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The Population of Near-Earth Asteroids in Coorbital Motion with the Earth

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We obtain the size and orbital distributions of near-Earth asteroids (NEAs) that are expected to be in the 1 : 1 mean motion resonance with the Earth in a steady state scenario. We predict that the number of such objects with absolute magnitudes $H < 18$ and $H < 22$ is 0.65 ± 0.12 and 16.3 ± 3.0, respectively. We also map the distribution in the sky of these Earth coorbital NEAs and conclude that these objects are not easily observed as they are distributed over a large sky area and spend most of their time away from opposition where most of them are too faint to be detected.

Key Words: **asteroids, dynamics; resonances.**

1. INTRODUCTION

A well known example of coorbital motion in the Solar System is the so-called Trojan asteroids (around 10^3 objects identified up to now) which move in the vicinity of the lagrangian points located along Jupiter's orbit, i.e., either L_4 or L_5. These objects are in the 1 : 1 mean motion resonance[1] with Jupiter as the difference in mean longitudes librates (in this case around $\pm 60°$) which protects them against collisions with the planet.

The jovian Trojan asteroids are probably primordial as most of them are in orbits which have been stable for a time on the order of the Solar System's age (Levison *et al.* 1997). In this scenario, these bodies are residues from the time of Jupiter's formation that were left in coorbital motion with this planet. Surprisingly, no objects in coorbital motion with Saturn, Uranus, or Neptune have been found to date. This could be partly due to the difficulty in observing such distant objects and partly due to the effect of long-term n-body perturbations (Nesvorny *et al.* 2002, Morais 2001) and/or primordial mechanisms (Gomes 1998).

There are currently two confirmed objects in the 1 : 1 mean motion resonance with Mars: (5261) Eureka and 1998 VF31. Additionally, a member of the population of near-Earth asteroids (NEAs), (3753) Cruithne (previously known as 1986 TO), was recently shown to be in the 1 : 1 mean motion resonance with the Earth (Wiegert and Innanen 1998). The orbit of this object is very eccentric ($e = 0.515$, while in the case of the Earth $(\mu/3)^{1/3} = 0.01$); such a high value of the orbital eccentricity implies that the motion of Cruithne is very different from that of the Trojan asteroids (in particular, its orbit is not at all similar to that of the Earth). The peculiar motion of Cruithne motivated new studies on the dynamics of the 1 : 1 mean motion resonance by Namouni (1999).

Other examples of captures in the 1 : 1 mean motion resonance with either the Earth or Venus have been found in numerical integrations of NEAs. Christou (2000) showed that (10563) Izhdubar (previously known as 1993 WD), (3362) Khufu, and 1994 TF2 could become Earth coorbitals, while 1989 VA could become a Venus coorbital. On the other hand, Michel (1997) followed a clone of (4660) Nereus that is captured in the 1 : 1 mean motion resonance with Venus.

The violent nature of the currently believed scenario for the formation of the terrestrial planets (Chambers and Wetherill 1998) is not favorable to the existence of a significant primordial population of Trojans asteroids associated with them. While recent numerical integrations up to 100 Myr of martian Trojans showed that both (5261) Eureka and 1998 VF31

[1] An object can be in the 1 : 1 mean motion resonance with a given planet only if $|a - a_1| < (\mu/3)^{1/3} a_1$, where μ is the planet-Sun mass ratio, and a and a_1 are respectively the object's and planet's orbital semimajor axes. In addition, if the orbital eccentricity is reasonably small as in the case of the Trojan asteroids, i.e., if $e \lesssim (\mu/3)^{1/3}$, then the object has an orbit very similar to that of the planet. This fact motivated the use of the term coorbital, actually first used in the context of the 1 : 1 mean motion resonance between the saturnian satellites Janus and Epimetheus. On the other hand, if the orbital eccentricity is significant, i.e., $e \gg (\mu/3)^{1/3}$, then the object has an orbit very different from that of the planet. Nevertheless, in what follows, we use the term coorbital to refer to objects in the 1 : 1 mean motion resonance with a given planet, irrespective of the magnitude of their orbital eccentricity.

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lie in regions that are apparently stable (Evans and Tabachnik 2000a), this integration time is a mere 2% of the Solar System's age. On the other hand, although numerical integrations up to 50 Myr of fictitious Earth Trojans end with some surviving objects (Evans and Tabachnik 2000a), none of these has been identified to date.

Only recently has a major consensus been reached regarding the origin of NEAs (see review in Morbidelli *et al.* 2002a). It is now believed that these objects are being constantly injected into certain regions in the main asteroid belt where the orbital eccentricities can grow up to Earth crossing (or at least near-Earth crossing) values. Simulations show that NEAs have lifetimes on the order of 10 Myr and that they have very chaotic orbits with Lyapunov times of a few hundred years (Tancredi 1998). Typical removal mechanisms are collision with the Sun, a close approach to Jupiter followed by ejection onto an hyperbolic orbit, or (less frequent) collision with a terrestrial planet. Evidence from the crater record on the Earth–Moon system suggests that the population of NEAs has been in a steady state for the last 3 Gyr (Grieve and Shoemaker 1994).

The identification of NEAs is of utmost practical importance as some of these objects could be on a route of collision with the Earth. The need to optimize observational searches for these objects led to the recent development of a model for the orbital and size distribution of NEAs (Bottke *et al.* 2000, Bottke *et al.* 2002). This model is based on two assumptions: (1) the absolute magnitude (H) distribution obeys a single parameter (source-independent) law valid for $13 < H < 22$; (2) the NEA population is being supplied in a steady state by main belt sources. The orbital evolution of thousands of test bodies initially located at the various main belt sources is then followed as they become NEAs. This procedure allows the definition of the residence time probability distributions of NEAs coming from each main belt source as a function of semimajor axis (a), eccentricity (e), and inclination (I). A model of the distribution of NEAs is then constructed by taking a linear combination of the orbital distributions from each source together with the one-parameter law for the absolute magnitude distribution. Subsequent comparison with the available data, while taking into account the observational biases, allows the determination of the best fit parameters of this model (i.e., the parameter of the size distribution and the fluxes from each main belt source which are necessary to keep the NEA population in a steady state).

In this paper we test the hypothesis of the existence of a population of Earth coorbital NEAs, which are constantly being supplied by main belt sources. Using as a starting point the predictions of the NEA model of Bottke *et al.* (2002),[2] we estimate the steady state number (according to size) of Earth coorbital NEAs, we obtain their orbital elements' distribution, and finally we investigate their observability.

[2] This is, in fact, a model for the size and orbital distributions of near-Earth objects (NEOs) that include not only the NEAs but also the near-ecliptic comets (NECs). However, as according to this model, near $a = 1$ AU the number of NECs is negligible compared to the number of NEAs, in this paper we just refer to the NEAs.

2. METHODOLOGY

The existent NEA model cannot currently provide us with estimates for the steady state number (according to size) of Earth coorbital NEAs nor for their typical orbital parameters. This is due to the fact that this model was constructed on the basis of numerical integrations whose output was sampled at 10,000-year intervals, which is much larger than a typical synodic period[3] and in fact on the order of the duration of a coorbital episode for Earth coorbital NEAs (Christou 2000). In addition, capture of a NEA in the 1 : 1 mean motion resonance with the Earth should be a rare event and therefore the existent NEA model would, in any case, provide us with low number (and therefore unreliable) statistics.

In order to achieve our goal of estimating the population of Earth coorbital NEAs, we followed the evolution of test bodies with e and I chosen according to the existent NEA model for $1.1\ \text{AU} < a < 1.2\ \text{AU}$ (our intermediate source region) as they became coorbital with the Earth (our target region) and until they collide with the Sun or a terrestrial planet or achieve $a > 10$ AU (at which point it is very unlikely that they will return to the inner Solar System). The orbital elements of these test bodies (as well as those of the planets) were outputted every 100 years as a compromise between being able to accurately identify coorbital motion with the Earth and generating manageable amounts of data.

In order to decide if a test body is captured in the 1 : 1 mean motion resonance with the Earth, we checked for oscillation of its semimajor axis around 1 AU. Oscillations with periodicity greater than 10,000 years (thus much larger than a typical synodic period; see above) or that happened for less than 10 consecutive output times were rejected.

We stress that the setting of the intermediate source (IS) as the NEA region with $1.1\ \text{AU} < a < 1.2\ \text{AU}$ is correct, as any test bodies coming from the main belt will enter this region before becoming Earth coorbitals and are also likely to remain in this region for a time longer than 10,000 years (the interval at which the output of the numerical integrations is sampled in the NEA model) with the exception of those rare cases in which they jump over the IS region due to a deep planetary close encounter. The advantage of this choice of IS region is that, in this way, we increase the likelihood of a test body being captured as an Earth coorbital (because we start with a large population in a region already quite close to the target region) and can therefore obtain more accurate statistics than if we followed the test bodies all the way from the main belt sources. Obviously, we could not follow this methodology if we did not yet have a prediction of the NEAs' orbital and size distributions in the region with $1.1\ \text{AU} < a < 1.2\ \text{AU}$.

[3] We can use, for instance, the synodic period of a tadpole orbit which is equal to $[(27/4)\mu]^{-1/2}$ orbital periods (Message 1966): for an Earth–Sun mass ratio $\mu = 3 \times 10^{-6}$, the synodic period is then $\sim$200 years.

Now, if we assume a steady state scenario, we can estimate the population in the target (TR) region, which is

$$N_{\mathrm{TR}} = F \times L_{\mathrm{TR}}, \tag{1}$$

once we know F (the flux of entrance in the TR region) and L_{TR} (the mean lifetime in the TR region).

The quantity L_{TR} can be obtained directly from the simulations. In order to compute F, we make use of the fact that our simulated situation corresponds to a steady state scenario in which we suddenly stop feeding the IS region. Therefore, the subpopulation of the IS region that feeds the TR region starts decaying into the TR region, at a rate

$$\frac{\mathrm{d}N_{\mathrm{s}}}{\mathrm{d}t} = -r_{\mathrm{IS}}(t) N_{\mathrm{s}} \tag{2}$$

where $r_{\mathrm{IS}}(t)$ is the fractional decay rate into the TR region.

Now, if t is sufficiently small, the fractional decay rate can be approximated by a constant value, r_{IS}. Hence, the number of bodies in the IS region that still have not entered the TR region at time t is

$$N(t) = N_{\mathrm{s}}(t) + N_{\mathrm{IS}} - f N_{\mathrm{IS}}, \tag{3}$$

where

$$N_{\mathrm{s}}(t) = f N_{\mathrm{IS}} \exp[-r_{\mathrm{IS}} t] \tag{4}$$

is the number of bodies that belong to the subpopulation of the IS region that feeds the TR region and that are still in the IS region at time t, N_{IS} is the initial population in the IS region, and f is the fraction of test bodies that enter the TR region at some stage (hence $f N_{\mathrm{IS}}$ is the total number of bodies that belong to the subpopulation of the IS region that feeds the TR region).

Moreover, if the chosen initial conditions are representative of the steady state orbital distribution in the IS region, then the flux of entrance in the TR region is given by

$$F = r_{\mathrm{IS}} f N_{\mathrm{IS}}; \tag{5}$$

hence we can obtain F once we know f and r_{IS}. As these are two independent variables in Eq. (4) it is better to use Eq. (3). In fact, by expanding the exponential in Eq. (4) in a Taylor series around $t = 0$, we can show that Eq. (3) can be rewritten as

$$N(t) = N_{\mathrm{IS}} \exp[-r_{\mathrm{IS}} f t] \tag{6}$$

and therefore we can obtain the quantity $r_{\mathrm{IS}} f$ by fitting a straight line to $\ln[N(t)]$.

3. RESULTS AND DISCUSSION

3.1. The Numerical Integration Scheme

All our numerical integrations were made using the "swift-rmvs3" integrator (Levison and Duncan (1994)), a modification of the symplectic algorithm proposed by Wisdom and Holman (1991) which is able to deal with planetary close encounters. In order to test its accuracy when applied to our problem, we conducted two sets of tests. First, we checked for the conservation (over a 1-Myr timescale) of the Jacobi integral and of the Kozai integral of the secular motion (Namouni 1999) when using a time step of 4 days to follow the orbits of coorbital objects with high e and I, in the framework of the circular restricted three-body problem with an Earth–Sun mass ratio. Subsequently, we followed 25 clones of asteroid Cruithne (generated as in Christou (2000), by varying the initial conditions within the observational uncertainties) for 0.1 Myr with time steps of 0.25, 1, and 4 days, while including the gravitational effects of the eight planets Mercury to Neptune. In Fig. 1 we show the cumulative probability distribution for the time, from the present date, spent by Cruithne as an Earth coorbital. We can see that this does not appear to change significantly with the choice of time step; hence we decided to use "swift-rmvs3" with a time step of 4 days (a value slightly more conservative than the time steps of 15 and 7.5 days used in previous NEA studies (e.g., Migliorini *et al.* 1997, Gladman *et al.* 1997)).

3.2. Statistics from the Numerical Integrations

We followed a set of 1900 test bodies initially in the IS region as their orbits evolve subject to gravitational perturbations from the seven planets Venus to Neptune. This set was followed first for 5 Myr, in order to estimate the flux of entrance in the TR region.

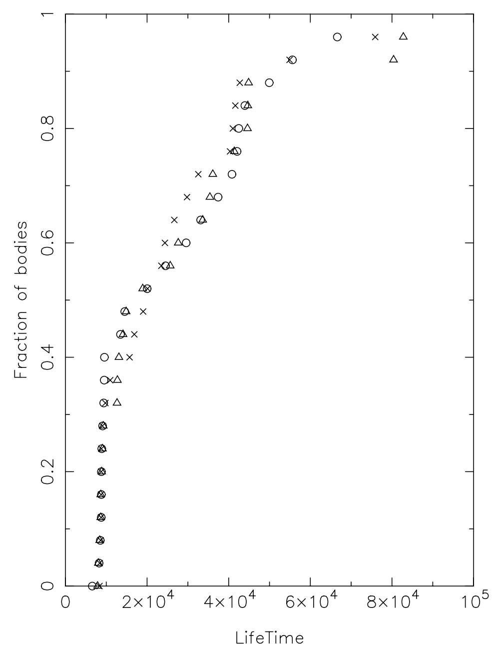

FIG. 1. The cumulative probability distribution for the time (from the present date) spent by Cruithne as an Earth coorbital. Different symbols refer to numerical integrations using various time steps: 0.25 day (crosses); 1 day (triangles); 4 days (circles).

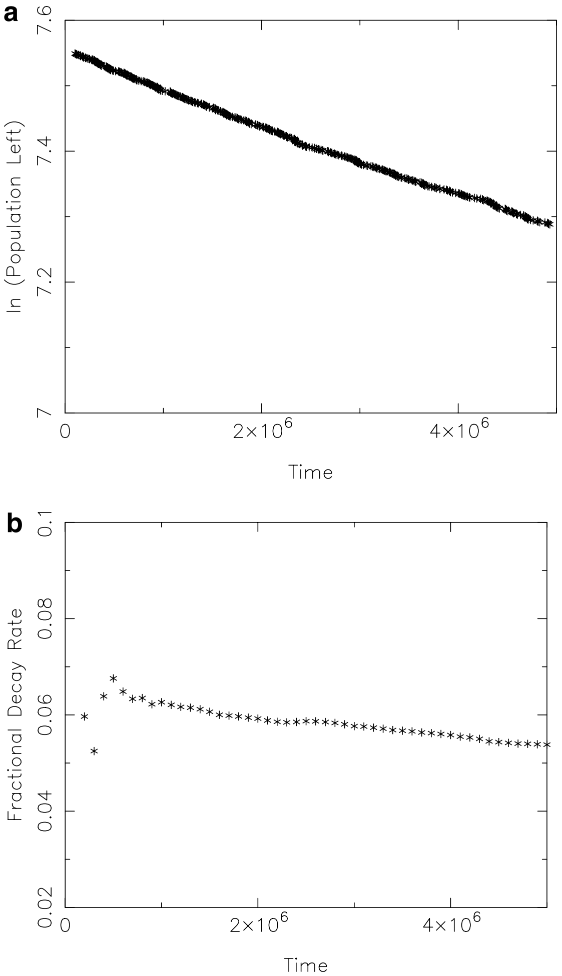

FIG. 2. The decay rate from the IS region into the TR region: (a) logarithm of the population left in the IS region as a function of time; (b) fractional decay rate into the TR region obtained by fitting a straight line to the graphic (a) in the interval $[0, T]$.

Now, according to Eq. (5), the flux of entrance in the TR region depends on f (the fraction of test bodies in the IS region that feed the TR region), $r_{\rm IS}$ (the constant fractional decay rate into the TR region), and $N_{\rm IS}$ (the steady state number of NEAs in the IS region).

The quantity $r_{\rm IS} f$ is, according to Eq. (6), the absolute value of the slope of $\ln[N(t)]$. In Fig. 2a we show the plot of $\ln[N(t)]$, and in Fig. 2b we show the plot of $r_{\rm IS} f$, obtained by fitting a straight line by the method of linear regression to $\ln[N(t)]$ in the interval $[0, T]$, with T varying from 0.2 to 5 Myr at 0.1-Myr steps. Now, the computation of the fractional decay rate for too small T is unreliable due to the small number of data points available. On the other hand, for T too large we are attempting to fit a straight line to a part of $\ln[N(t)]$ where the number of test bodies left in the subpopulation of the IS region that feeds the TR region is too small and consequently the decay rate deviates from an exponential law. By inspection of Fig. 2b we decided to compute an average fractional decay rate by including only the values between $T = 0.4$ and $T = 1.3$ Myr, which gives $r_{\rm IS} f = -0.0633 \pm 0.0013$ Myr^{-1}.

Finally, from the NEA model (Bottke *et al.* 2000, Bottke *et al.* 2002) we can obtain the steady state number of NEAs with $H < H_0$ in the IS region. In particular, $N_{\rm IS} = 31 \pm 6$ for $H_0 = 18$ and this value scales approximately as $10^{0.35(H_0-18)}$ for $18 < H_0 < 22$. We can then apply Eq. (5) to obtain the flux of entrance in the TR region: this is $F = 1.96 \pm 0.38$ test bodies Myr^{-1} for $H < 18$.

Now, according to Eq. (1), in order to obtain the steady state number of Earth coorbital NEAs we need to compute $L_{\rm TR}$ (the mean lifetime in the target region). In order to obtain this quantity, we followed up to 100 Myr the set of test bodies that after 5 Myr had at some stage been coorbital with the Earth (438 out of 1900). This integration timespan proved to be long enough for our purposes, as at the end of it, not only were there very few surviving test bodies but new captures in the 1 : 1 mean motion resonance were very unlikely (indeed, at the end of the 100-Myr timespan the cumulative time spent in the TR region had almost reached a plateau). We obtained a mean lifetime in the 1 : 1 mean motion resonance with the Earth, $L_{\rm TR} = 0.33$ Myr. Applying Eq. (1) we can then predict the steady state number of Earth coorbital NEAs: this is $N_{\rm TR} = 0.65 \pm 0.12$ and $N_{\rm TR} = 16.3 \pm 3.0$ for $H < 18$ and $H < 22$, respectively.

An interesting by-product of our simulations was the realization that typically a test body experiences several captures/escapes in/from the 1 : 1 mean motion resonance: the average duration of a coorbital episode was 25,000 years (while none lasted longer than 1 Myr) and consecutive coorbital episodes were spaced on average by 2 Myr. This had not been realized before, probably because the few existent numerical integrations of objects in the 1 : 1 mean motion resonance with the Earth had been extended at most up to 200,000 years (e.g., Christou 2000).

3.3. Comparison with the Current Observations

The only NEA which is currently known to be in the 1 : 1 mean motion with the Earth, (3753) Cruithne, has absolute magnitude[4] $H = 15.10$. In addition, according to Christou (2000), three other NEAs can become Earth coorbitals in the timespan of 0.2 Myr centered at the present time; these are (10563) Izhdubar, (3362) Khufu, and 1994 TF2, with $H = 16.90$, $H = 18.30$, and $H = 19.30$, respectively.

The numerical integrations of Christou (2000) show that 100% of the clones of Cruithne, and about 50% of those of Izhdubar and Khufu, were at some stage in the 1 : 1 mean motion resonance with the Earth during the timespan $\Delta t = 0.2$ Myr. Thus we conclude that, on average, two of the three NEAs which have $H \lesssim 18$ should become Earth coorbitals within this timespan.

From the results of the previous section we know that the steady state number of Earth coorbital NEAs with $H < 18.30$

[4] Absolute magnitudes, H, of near-Earth asteroids are available at http://earn.dlr.de/nea.

is $N_{TR} < (0.65 \pm 0.12) \times 10^{0.35 \times 0.3} = 0.83 \pm 0.15$. As this is of course a mean value, ideally we would like to compare it with the number of NEAs which are in the 1 : 1 mean motion resonance with the Earth over a large enough timespan. Therefore, we chose to compare our results to those from the numerical integrations for $\Delta t = 0.2$ Myr performed by Christou (2000).

The number of NEAs that will become Earth coorbitals within a given timespan, Δt, will depend on the time, $\Delta t_{1:1}$, that they are likely to spend in the 1 : 1 mean motion resonance with our planet within this timespan. Now, as the average time interval between consecutive coorbital episodes is 2 Myr, there is on average one coorbital episode per coorbital clone in the timespan $\Delta t = 0.2$ Myr. Therefore, we can then set $\Delta t_{1:1} = 25{,}000$ years (which is the average duration of a coorbital episode in our simulations), and the number of NEAs with $H < 18.3$ that should become Earth coorbitals within Δt is then $(0.83 \pm 0.15) \times \Delta t / \Delta t_{1:1} = 6.6 \pm 1.2$. Now, if we assume that Christou (2000) identified all the known NEAs with $H \lesssim 18$ that will become Earth coorbitals during $\Delta t = 0.2$ Myr, we conclude that, at the time of his search, the observations of objects with $H < 18.3$ near $a = 1$ AU had completeness between 26 and 37% which is consistent with the predictions of the NEA model of Bottke *et al.* (2002).

3.4. Types of Coorbital Modes

The binned eccentricity and inclination distributions, obtained while the test bodies were in the 1 : 1 mean motion resonance with the Earth, are shown in Figs. 3a and 3b, respectively. We see that the typical eccentricities are quite high, that most objects have orbits which are Venus crossing ($e \geq 0.28$), and that many have orbits which are Mars crossing ($e \geq 0.52$). In fact, the evolution of these orbits seems to be mostly caused by close encounters with the terrestrial planets (Christou 2000) which is typical of NEAs. On the other hand, the role played by secular resonances inside the coorbital region (Morais 2001) could also be important in the case of long-lived coorbital episodes as seen by Michel (1997) in the case of Venus. In Fig. 3b we see that the typical inclinations can also be quite high and that most objects have orbits with $10° < I < 45°$.

The dynamics in the 1 : 1 mean motion resonance at high values of the eccentricities and inclinations has been studied in the context of the three-body problem by Namouni (1999), Namouni *et al.* (2000), Namouni and Murray (2000), and Nesvorny *et al.* (2002). In brief, in the low-eccentricity regime, objects in the 1 : 1 mean motion resonance with a given planet which share the same orbital plane with it (planar problem) can be in either tadpole modes (the critical angle librates around either 60 or $-60°$) or horseshoe modes (the critical angle librates around $180°$ and the libration encloses $\pm 60°$). In the high-eccentricity regime, there are also retrograde satellite modes (the critical angle librates around $0°$) and libration in the tadpole and horseshoe modes does not necessarily enclose $\pm 60°$ but rather points which are displaced towards $180°$ by an amount proportional to the eccentricity. In the three-dimensional problem there are also stable compound modes (for instance, combinations of tadpole or horseshoe modes with retrograde satellite modes) and transitions between different coorbital modes are possible on a secular timescale.

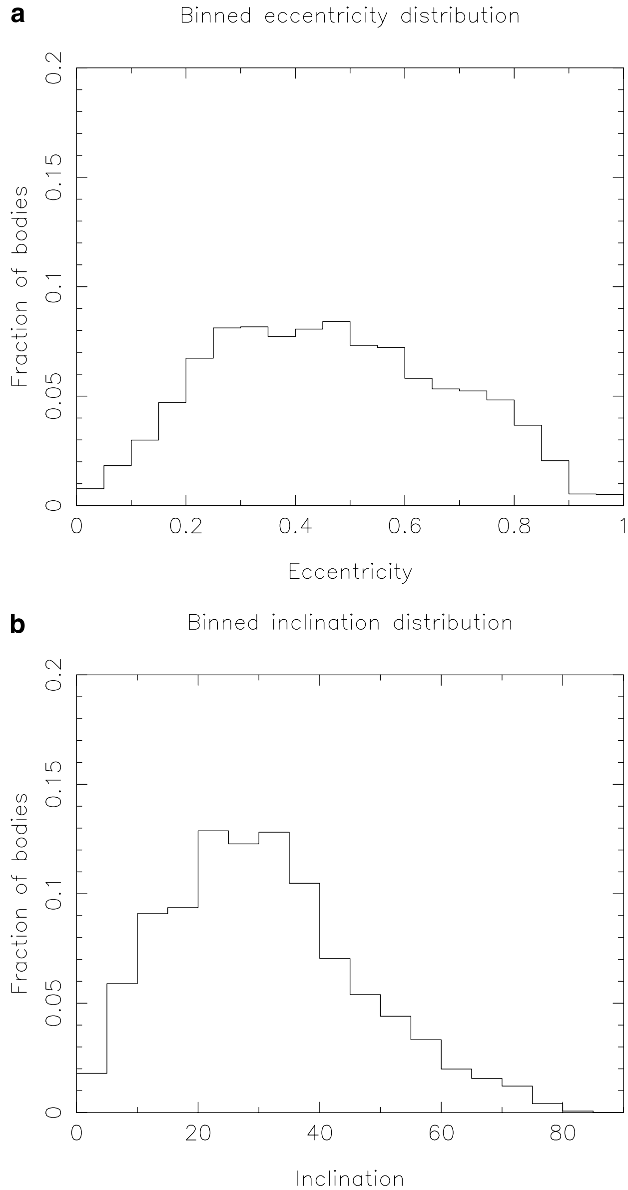

FIG. 3. The binned eccentricity (a) and inclination (b) distributions for the Earth coorbitals from our simulations.

3.5. The Distribution of Earth Coorbitals in the Sky

In order to obtain the distribution in the sky of Earth coorbitals, we first computed the time spent by these in each $5 \times 5°$ cell in ecliptic longitude and latitude coordinates and then normalized this by the total time spent in the 1 : 1 mean motion resonance. This residence-time probability distribution of Earth coorbital NEAs is depicted in Fig. 4a in geocentric ecliptic coordinates.

The observability of a given object depends on its apparent visible magnitude which, according to Bowell *et al.* (1989), is

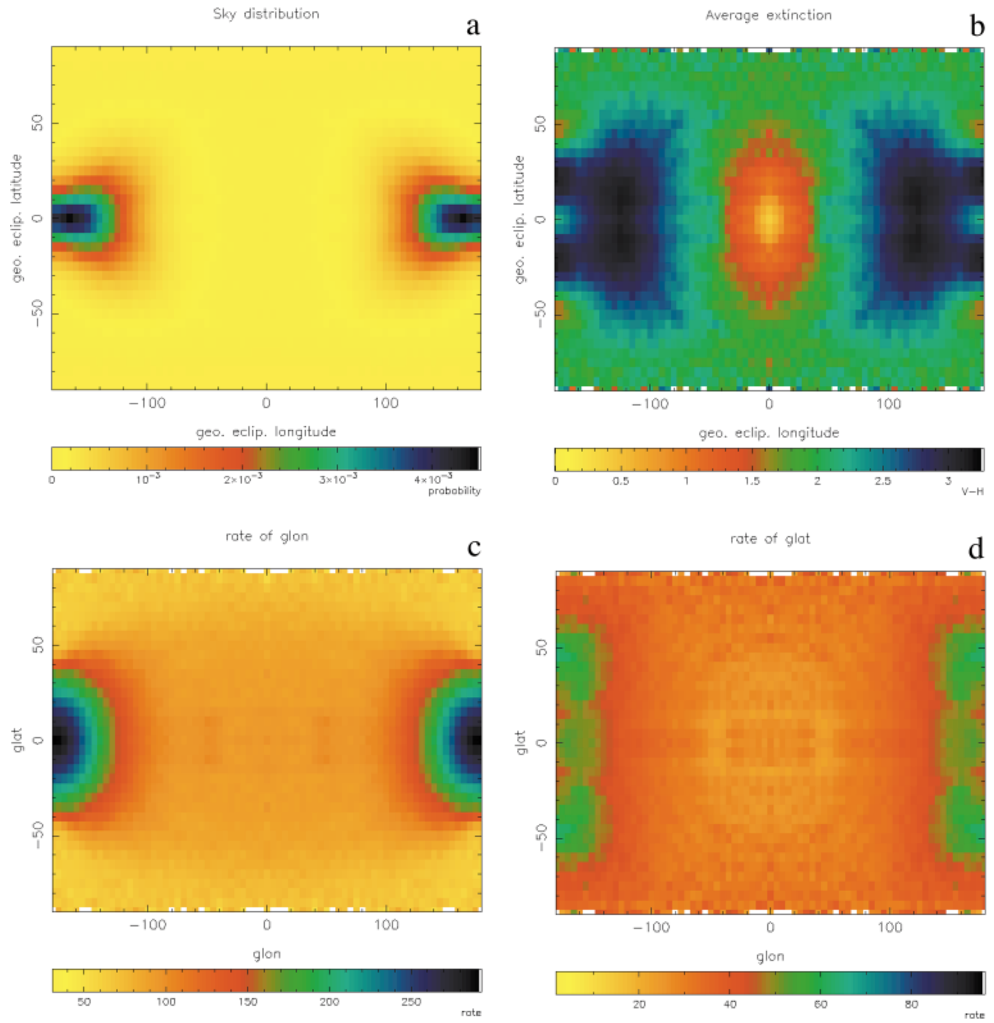

FIG. 4. Representation in geocentric ecliptic coordinates of several characteristics of the distribution of Earth coorbitals. The color in each $5 \times 5°$ cell indicates for these objects: (a) the probability of finding one of them; (b) the average extinction; (c) the average longitudinal rate of motion, and (d) the average latitudinal rate of motion (both rates measured in$''$ h^{-1}). The opposition has geocentric ecliptic latitude and longitude 0°.

given by

$$V = H - 2.5\log[(1-G)\Phi_1(\alpha) + G\Phi_2(\alpha)] + 5\log[r \times \Delta] \tag{7}$$

where H is the absolute magnitude, α is the phase angle, G is termed the slope parameter and is related to the asteroid's albedo, and r and Δ are respectively the distances to the Sun and Earth; $\Phi_1(\alpha)$ and $\Phi_2(\alpha)$ are defined in Eq. (A5) of Bowell *et al.* (1989).

The extinction,[5] $V - H$, provides a measure of the difference in brightness with respect to an intrinsic value, H, the latter being a function of the size and albedo of the object being observed. This quantity, $V - H$, varies according to the object's albedo (due to the dependence on G) and to its distance and relative position with respect to the Earth and Sun (due to the dependence on r, Δ, and α). In Fig. 4b we show a representation of the average extinction for the Earth coorbitals from our simulation computed using a slope parameter $G = 0.21$. This value was obtained by taking the weighted mean value between S-type and C-type asteroids, which according to E. F. Tedesco (personal communication, 2001) have respectively $G = 0.23$ and $G = 0.12$, while assuming that NEAs with H smaller than a given value are a mixture of 80% S types and 20% C types (Morbidelli *et al.* 2002b). The extinction depicted in Fig. 4b was obtained by computing the average of the individual extinctions for each output step spent by a coorbital test body within each $5 \times 5°$ cell in ecliptic longitude and latitude coordinates.

In Fig. 4a we see that the higher concentration of Earth coorbital NEAs occurs for geocentric ecliptic longitudes in the

[5] Here, the term extinction refers to the difference in brightness, $V - H$, which is a function of the position of the asteroid with respect to the Sun and the Earth. It does not include the effect of the atmospheric extinction which is a function of the zenithal angle of the object being observed; see below.

vicinity of the Sun. Unfortunately, this region of the sky is inaccessible from the Earth. In fact, typically observations are possible only when the Sun is at least 18° below the horizon and when the air mass is less than 2 (when the air mass exceeds this value, the atmospheric extinction is often too great), i.e., when the object is at least 30° above the horizon. This implies that usually objects can only be observed when they have an ecliptic longitude that differs from that of the Sun by an amount that must be at least 48° in absolute value.

Taking this effect into account and by inspecting Fig. 4a, it seems reasonable at first glance to propose that observational searches be made at ecliptic latitudes between 0 and ±10° and ecliptic longitudes relative to opposition between 120 and 130° (or −120 and −130°). As each of these two regions is composed of 8 cells of 5 × 5°, in which the probability density is around 0.0024, then the probability of finding in each one of them an object with $H < H_0$ is 0.0192 N_{TR}, where N_{TR} is the number of NEAs with $H < H_0$. This gives probabilities of $1.2 \pm 0.2\%$ and $31 \pm 6\%$ for $H_0 = 18$ and $H_0 = 22$, respectively.

Now, by comparing Figs. 4a and 4b, we see that in the region that we proposed be searched, the average extinction is around 3. Therefore, in order to detect NEAs with $H < 22$ we would need a telescope with limiting magnitude 25. Searching a $20 \times 10°$ sky area at such a limiting magnitude is certainly not feasible in practice. If we assume that we could conduct such a wide-field survey at limiting magnitude 24 (as in the ambitious Large Synoptic Survey Telescope (LSST) project (http://dmtelescope.org/index.htm)), then due to the extinction effect we would be limited to search for objects with $H < 21$ which have a probability of merely $14 \pm 2.5\%$ of being found in each of these two regions, and therefore even this would be quite an inefficient survey.

In Fig. 4c we show the average rate of geocentric ecliptic longitude motion as a function of geocentric ecliptic coordinates. We see that this takes values between 100″ h^{-1} (at opposition) and 300″ h^{-1} (at 180° from opposition). In Fig. 4d we show the average absolute value of the rate of geocentric ecliptic latitude motion, again as a function of geocentric ecliptic coordinates.

In the region with ecliptic latitudes between 0 and ±10° and ecliptic longitudes relative to opposition between ±120 and ±130°, we see that the rate of geocentric ecliptic longitude motion ranges from about 150 to 160″ h^{-1}, i.e., it is not so different from the Earth's mean motion (148″ h^{-1}) as we would expect in the case of small eccentricity orbits in the vicinity of L_4 or L_5. On the other hand, the absolute value of the rate of ecliptic latitude motion is around 40 or 45″ h^{-1} which is consistent with the existence of inclined orbits at this location.

Such fast rates of motion make detection of faint objects difficult, as during the exposure times required to collect a signal the objects can cross the seeing disk of the image. This problem can in principle be avoided by tracking the telescope at the object's expected rate of motion, as done by Whiteley and Tholen (1998) who tried to find Earth Trojan asteroids by covering a 0.35-square-degree sky area around the L_4 and L_5 points of the Earth–Sun system while tracking the telescope at 150″ h^{-1} along

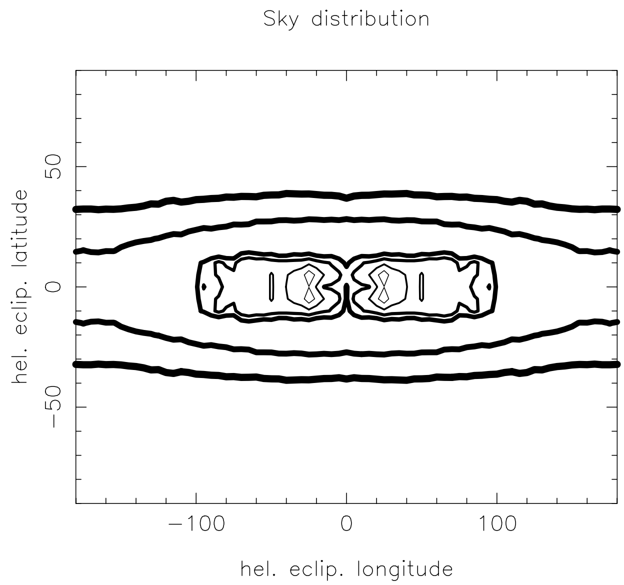

FIG. 5. A contour plot representation in heliocentric ecliptic coordinates of the residence-time probability distribution of Earth coorbitals: the contours taken at 90, 85, 80, 75, 50, and 25% are represented by lines of increasing thickness. The Earth has heliocentric ecliptic latitude and longitude 0°.

the ecliptic. This technique is useful if the latitudinal motion is nearly zero (which is a reasonable assumption when searching for a primordial population of Trojans in the vicinity of L_4 or L_5). However, in our case the latitudinal motion can be quite fast and this effect can pose additional difficulties for the detection of faint Earth coorbital NEAs by reducing the effective limiting magnitude of the survey.

In Fig. 5 we show a contour plot in heliocentric ecliptic coordinates of the residence-time probability distribution of Earth coorbital NEAs. We see that the distribution of ecliptic latitudes is broad and in particular the 25% contour is in between ecliptic latitudes ±30 and ±40° which is explained by the existence of a significant number of orbits with inclinations up to 40°, in agreement with Fig. 3b.

It is instructive to compare the residence-time probability distribution depicted in Fig. 5 with what we would expect from a hypothetical primordial population of Earth Trojan asteroids, a subject which was investigated by Wiegert *et al.* (2000) and by Evans and Tabachnik (2000b). In the latter cases the highest concentration should occur at zero heliocentric ecliptic latitude and a heliocentric ecliptic longitude relative to the Earth of ±60°, as we would expect from a population with low-eccentricity orbits and semimajor axes uniformly distributed inside the Earth's coorbital region, and from the fact that in the low-eccentricity regime all librations in the 1 : 1 mean motion resonance enclose the stationary points L_4 and L_5. In our case, owing to the high values of the eccentricities, not only are there no L_4 nor L_5 stationary solutions, but the libration centers for tadpole orbits are displaced towards 180° by an amount which increases with the orbital eccentricity, as shown by Namouni and Murray (2000). On the other hand, as discussed above, at high eccentricity there are, in addition to tadpole and horseshoe modes, retrograde satellite and compound modes. Finally, our coorbital objects are NEAs which were temporarily captured; hence we do not expect to find a population with orbital semi major axes uniformly distributed inside the coorbital region. The combination of all these effects generates the complexity that can be seen in Fig. 5.

4. CONCLUSION

In this paper we obtained the number (according to size) and the orbital distribution of NEAs that are expected to be in the 1 : 1 mean motion resonance with the Earth in a steady state scenario. This work is based on the NEA model developed by Bottke *et al.* (2000, 2002). In short, we numerically integrated the evolution of NEAs initially located in a region close to the terrestrial coorbital region (whose size and orbital distribution we know a priori, from the NEA model) and monitored those that are trapped in the 1 : 1 mean motion resonance with the Earth.

We saw that NEAs which become coorbital with the Earth typically experience multiple coorbital episodes, each lasting on average 25,000 years and none lasting longer than 1 Myr. These multiple coorbital captures were not seen in the numerical integrations of just a few Earth coorbitals performed by Christou (2000) probably because these were extended only up to 200,000 years. We also saw that the distributions of eccentricities and inclinations of the NEAs while they are in the 1 : 1 mean motion resonance with the Earth have maxima at $e = 0.4$ and $I = 30^\circ$, respectively, which corresponds to the coorbital regime studied by Namouni (1999), Namouni *et al.* (2000), Namouni and Murray (2000), and Nesvorny *et al.* (2002).

Finally, we obtained the sky distribution of Earth coorbital NEAs and saw that these objects are spread over a large sky area which poses difficulties for observational searches. In addition, we saw that these objects are undetectable most of the time, either because they are too low over the horizon (or even below it) or because they are too faint, with an extinction which can be as high as $V - H = 3$. Current programs for detecting NEAs go down at most to limiting magnitude 21.5 (as in the case of Spacewatch; J. Larsen, personal communication, 2001) and therefore are only likely to detect Earth coorbital NEAs with $H < 18$. In any case, the existence of (3753) Cruithne implies that the survey for Earth coorbitals with $H < 18$ is, at the current time, probably already complete. On the other hand, we believe that smaller Earth coorbital NEAs will continue to be found,[6] although completing the survey for these objects will probably not be an easy task. Previous dedicated searches for hypothetical primordial Earth Trojan asteroids made by Whiteley and Tholen (1998) were able to go to limiting magnitude 22.8 but only covered tiny sky areas and therefore would not be efficient in discovering Earth coorbital NEAs.

[6] Since the first submission of this paper we did a search of the Asteroid Orbital Element Database (ftp://ftp.lowell.edu/pub/elgb/astorb.html) and found two more objects that have some chance of being captured in the 1 : 1 mean motion resonance with the Earth in less than 100,000 years from the present time: these are 1998 UP1 and 2000 WN10 with $H = 20.39$ and $H = 19.77$, respectively. On the other hand, a recent Minor Planet Electronic Circular (http://cfa-www.harvard.edu/iau/mpec/K02/K02A92.html) reported on an object, 2002 AA29, which has $H = 23.9$ and according to our integrations, seems to be in a temporary horseshoe orbit with the Earth.

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REFERENCES

Bottke, W. F., R. Jedicke, A. Morbidelli, J. M. Petit, and B. Gladman 2000. Understanding the distribution of near-Earth asteroids. *Science* **288**, 2190–2194.

Bottke, W. F., A. Morbidelli, R. Jedicke, J. M. Petit, H. Levison, P. Michel, and T. Metcalfe 2002. Debiased orbital and size distribution of the near-Earth objects. *Icarus* **156**, 399–433.

Bowell, E., B. Hapke, D. Domingue, K. Lumme, J. Peltoniemi, and A. W. Harris 1989. Application of photometric models to asteroids. In *Asteroids II* (R. P. Binzel, T. Gehrels, and M. S. Matthews, Eds.), pp. 524–556. Univ. of Arizona Press, Tucson.

Chambers, J. E., and G. W. Wetherill 1998. Making the terrestrial planets: N-body integrations of planetary embryos in three dimensions. *Icarus* **136**, 304–327.

Christou, A. A. 2000. A numerical survey of transient coorbitals of the terrestrial planets. *Icarus* **144**, 1–20.

Evans, N. W., and S. A. Tabachnik 2000a. Asteroids in the inner Solar System—I. Existence. *Mon. Not. R. Astron. Soc.* **319**, 63–79.

Evans, N. W., and S. A. Tabachnik 2000b. Asteroids in the inner Solar System—II. Observable properties. *Mon. Not. R. Astron. Soc.* **319**, 80–94.

Gladman, B., F. Migliorini, A. Morbidelli, V. Zappalà, P. Michel, A. Cellino, Ch. Froeschlé, H. Levison, M. Bailey, and M. Duncan 1997. Dynamical lifetimes of objects injected into asteroid belt resonances. *Science* **277**, 197–201.

Gomes, R. S. 1998. Dynamical effects of planetary migration on primordial Trojan-type asteroids. *Astron. J.* **116**, 2590–2597.

Grieve, R. A., and E. M. Shoemaker 1994. The record of past impacts on Earth. In *Hazards Due to Comets and Asteroids* (T. Gehrels and M. S. Matthews, Eds.), pp. 417–462. Univ. of Arizona Press, Tucson.

Levison, H. F., and M. Duncan 1994. The long-term behavior of short-period comets. *Icarus* **108**, 18–36.

Levison, H. F., E. M. Shoemaker, and C. S. Shoemaker 1997. Dynamical evolution of Jupiter's Trojan asteroids. *Nature* **385**, 42–44.

Message, J. 1966. The dominant features of the long-period librations of the Trojan minor planets. In *Space Science, Part 2: Lectures in Applied Mathematics* **6** (J. Barkley Rosser, Ed.), pp. 70–78.

Michel, P. 1997. Overlapping of secular resonances in a Venus horseshoe orbit. *Astron. Astrophys.* **328**, L5–8.

Migliorini, F., A. Morbidelli, V. Zappalá, B. Gladman, M. Bailey, and A. Cellino 1997. Vesta fragments from ν_6 and 3 : 1 resonances: Implications for V-type NEAs and HED meteorites. *Meteorit. Planet. Sci.* **32**, 906–913.

Morais, M. H. M. 2001. Hamiltonian formulation of the secular theory for Trojan-type motion. *Astron. Astrophys.* **369**, 677–689.

Morbidelli, A., W. Bottke, P. Michel, and Ch. Froeschlé 2002a. Origin and evolution of near-Earth asteroids. In *Asteroids III* (W. Bottke, A. Cellino, P. Paolicchi, R. Binzel, Eds.). Univ. of Arizona Press, Tucson. In press.

Morbidelli, A., R. Jedicke, W. F. Bottke, P. Michel, and E. F. Tedesco 2002b. From magnitudes to diameters: The albedo distribution of near-Earth objects and the Earth collision hazard. *Icarus*, in press.

Namouni, F. 1999. Secular interactions of coorbiting objects. *Icarus* **137**, 293–314.

Namouni, F., and C. D. Murray 2000. The effect of eccentricity and inclination on the motion near the Lagrangian points L_4 and L_5. *Celest. Mech.* **76**, 131–138.

Namouni, F., A. A. Christou, and C. D. Murray 2000. Coorbital dynamics at large eccentricity and inclination. *Phys. Rev. Lett.* **83**, 2506–2509.

Nesvorny, D. 2000. On the stability of co-orbital motion with Jupiter, Saturn, Uranus and Neptune. In *American Astronomical Society, DPS Meeting* **32**, 19.02.

Nesvorny, D., F. Thomas, S. Ferraz-Mello, and A. Morbidelli 2002. A perturbative treatment of the co-orbital motion. *Celest. Mech.* **82**, 323–361.

Tancredi, G. 1998. Chaotic dynamics of planet-encountering bodies. *Celest. Mech.* **70**, 181–200.

Whiteley, R. J., and D. J. Tholen 1998. A CCD search for Lagrangian asteroids of the Earth–Sun system. *Icarus* **136**, 154–167.

Wiegert, P. A., and K. A. Innanen 1998. An asteroidal companion to the Earth. *Nature* **387**, 685–686.

Wiegert, P. A., K. A. Innanen, and S. Mikkola 2000. Earth Trojans: A study in support of observational searches. *Icarus* **145**, 33–43.

Wisdom, J., and M. Holman 1991. Symplectic maps for the N-body problem. *Astron. J.* **102**, 1528–1538.