Experiments : particle dynamics in turbulent flows

Jean-François Pinton

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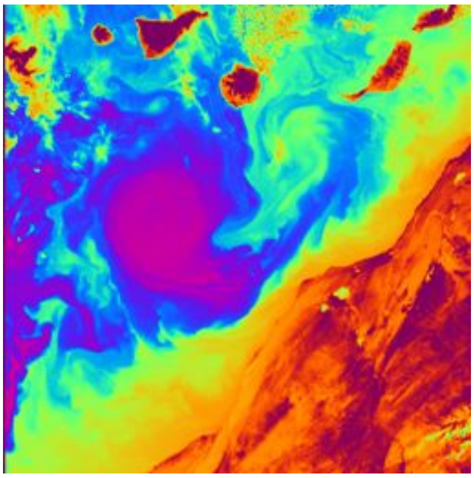
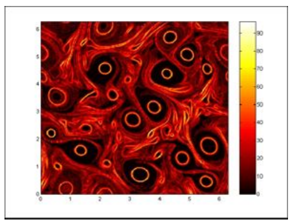
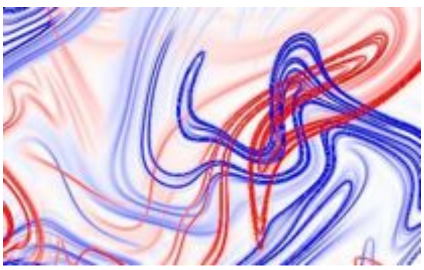
mixing







Turbulent dispersion

B. Sawford, *Ann. Rev. Fluid Mech.*, **33** (2001)

$$\partial_t C + \vec{u}.\vec{\nabla} C = \kappa \Delta C$$

- $\kappa = 0$

$$C(\vec{x},t) = \int_{s\leq t}\int_V d^3y dt\, p_1(\vec{x},t;\vec{y},s)\, S(\vec{y},s)$$

- $\kappa \neq 0$

Same equations at high Reynolds and Peclet numbers ,
Except very close to sources or boundaries
[Saffman, *JFM*, **8** (1960)]

Turbulent dispersion

B. Sawford, *Ann. Rev. Fluid Mech.*, **33** (2001)

$$\partial_t C + \vec{u}.\vec{\nabla} C = \kappa \Delta C$$

$$\frac{\partial P^{\mathscr{U}}(\boldsymbol{x},t|\boldsymbol{x}',t')}{\partial t} + \frac{\partial u_i P^{\mathscr{U}}(\boldsymbol{x},t|\boldsymbol{x}',t')}{\partial x_i} = \kappa \frac{\partial^2 P^{\mathscr{U}}(\boldsymbol{x},t|\boldsymbol{x}',t')}{\partial x_i^2}$$

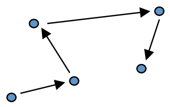
P^U is the Green's function of the scalar pb.

Turbulence, mixing and random walks

original motivation for Lag….

- Random walks

$$d\vec{V}(\vec{X},t) = -\gamma(\vec{V})dt \, + \, dG(t)$$



- White acceleration,

spectrum : $E_L^A(\omega) \propto \omega^0$

- Velocity spectrum,

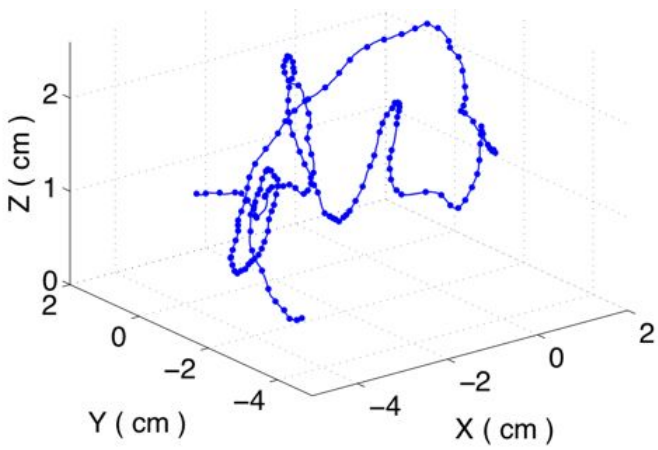
$$E_L^V(\omega) = C_0 \epsilon \, \omega^{-2}$$

dimensionally : $\langle v(t)v(t+\tau)\rangle_t = C_0(\epsilon\tau)$

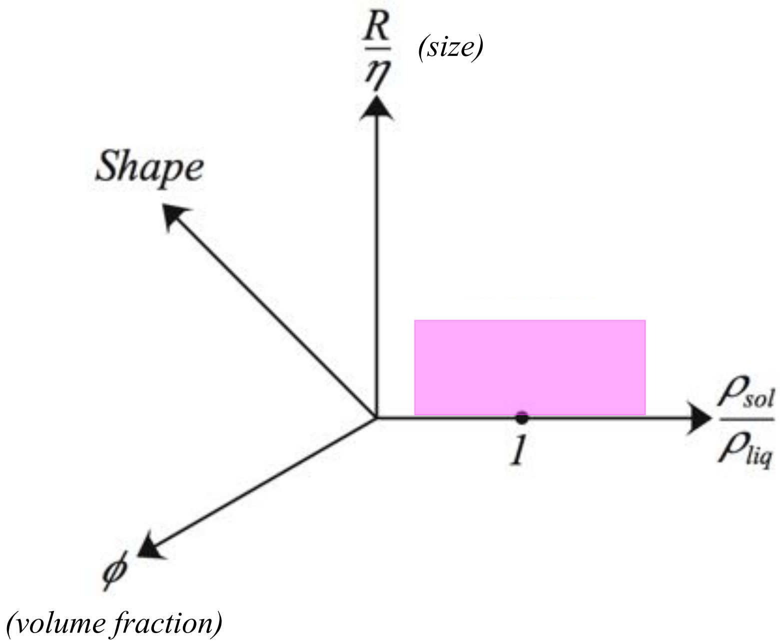
Euler & Lagrange

- Euler : **u**(**x**, t) , **x** in {flow domain}

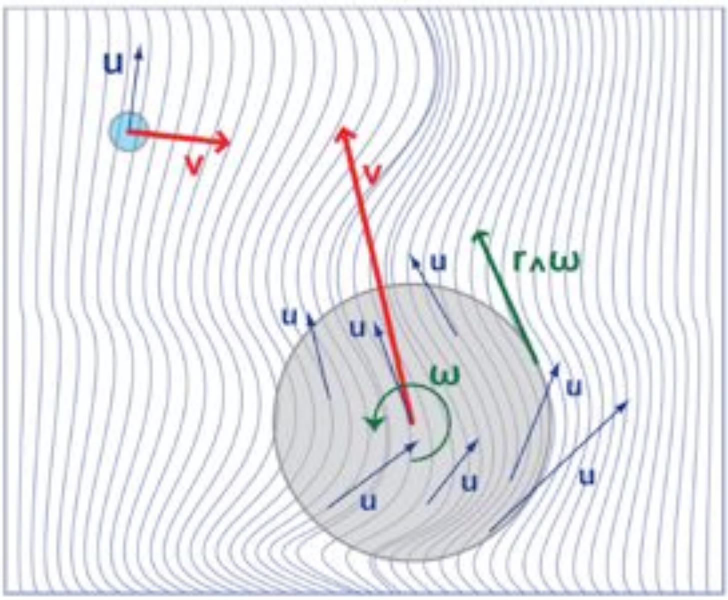
- Lagrange : **v**(**x**$_0$, t) , **x**$_0$ in {initial positions}



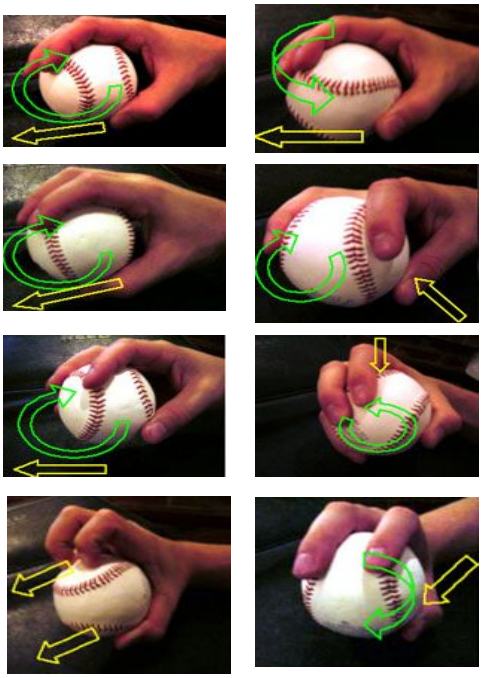
particle motions

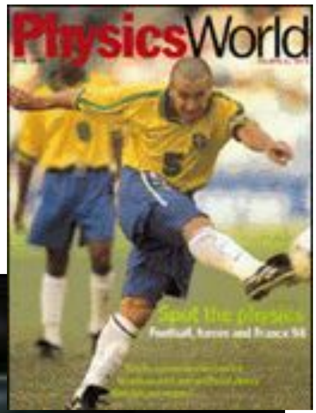


F = ma ... for geeks



turbulence mechanics 101





abstract

- Lecture 1:

  - Review of experimental techniques for lagrangian measurements,

  - The dynamics of tracers, single particle statistics.

- Lecture 2:

  - Issues and results associated with multiparticle statistics,

  - The dynamics of inertial particles: size effects and density effects.

some references

- B. Sawford, JFP, « A Lagrangian view of turbulent dispersion and mixing », Cambridge Lecture Notes, to appear.

- F. Toschi and E. Bodenschatz. Lagrangian Properties of particles in Turbulence. Ann Rev. Fluid Mech, 41 p 375 (2009)

abstract

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  - Review of experimental techniques for lagrangian measurements,

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  - Issues and results associated with multiparticle statistics,

  - The dynamics of inertial particles: size effects and density effects.

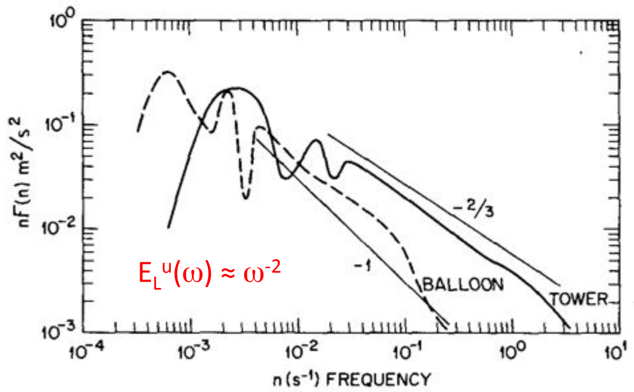
Experiments Gifford - Hanna

Gifford, *Month. Weath. Rev.*, **83**, 293, (1955)
Hanna, *J. Appl. Meteo.*, **20**, 242, (1981)

Neutral balloons + Doppler radar
- ratio $\beta = T_L / T_E$
- Re ≈ 25,000
- size $1m^3$
- sampling 1Hz, 1h runs

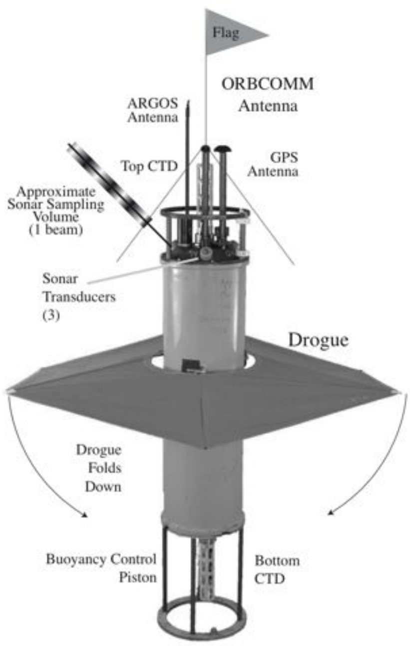
NATIONAL CENTER FOR ATMOSPHERIC RESEARCH (NCAR)
BOULDER, CO 80307-3000

Revision A May 1999

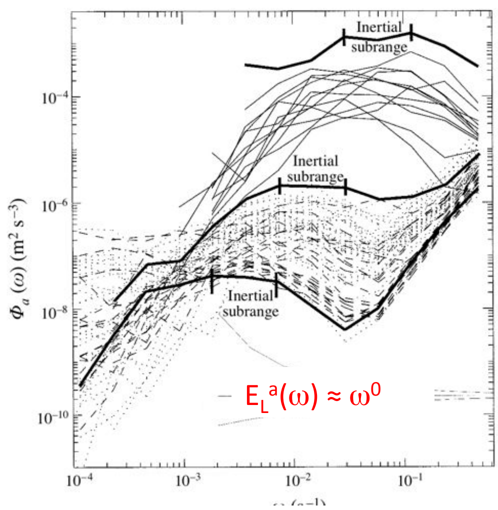


Experiments Lien D'Asaro

Lien, d'Asaro, Daikiri, *J. Fluid Mech.*, **362**, 177, (1998)



Floaters in ocean
- size 1.5m x 1.2m
- 1 Hz sampling
- 71 units



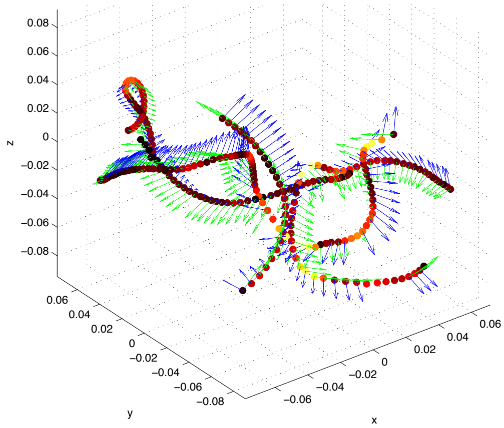
Resolved Particle Tracking

Fully developped turbulence, $R_\lambda \approx 1000$

- Scale resolution : $L/\eta \approx 10{,}000$

- Time resolution : $T/\tau_\eta \approx 1{,}000$

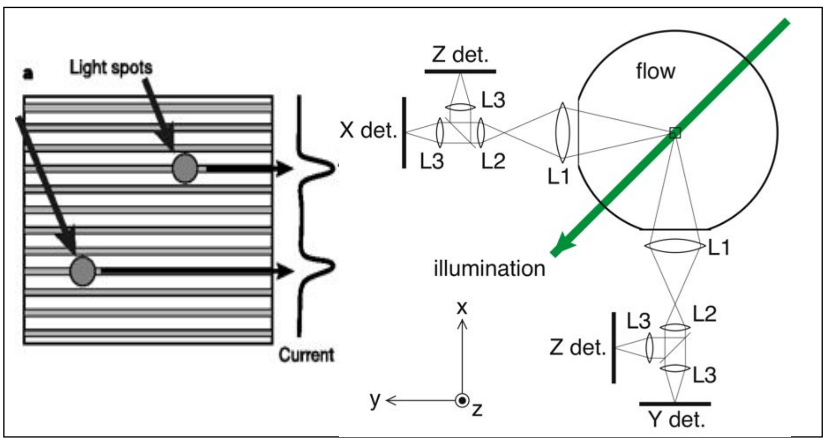
- Lab experiment : $L \approx 10$ cm - 100 cm
  : $T \approx 0.1$ s - 1 s
  pixel size ≈ 10 µm (NB: max 1024^2)
  sample rate > 10 kHz



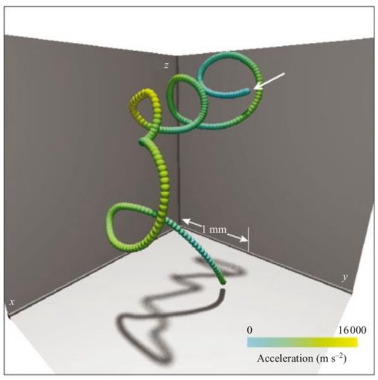
- Data size
  $N = (L/\eta)^2\,(T/\tau_\eta)\,(10) \approx 10^{12} = 1\text{Tb}/\text{s}$
  per video channel

Silicon-strip detectors

Voth, Satyanarayan, Bodenschatz, *Phys. Fluids*, **10**, 2268, (2000)



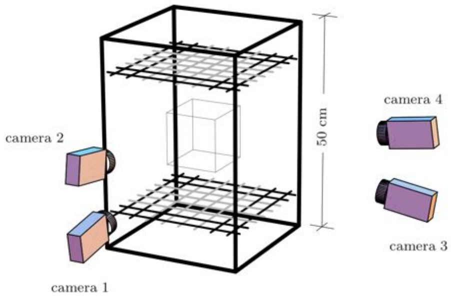
Particles : 10 µm
Pictures : 512 x 512 pixels
Rate : 70,000 images / sec
Record : 4000 images
$R_\lambda \approx 800$

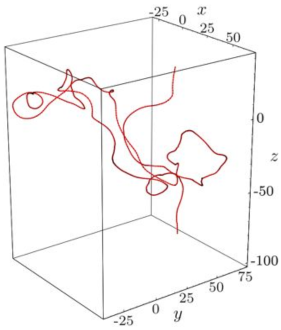


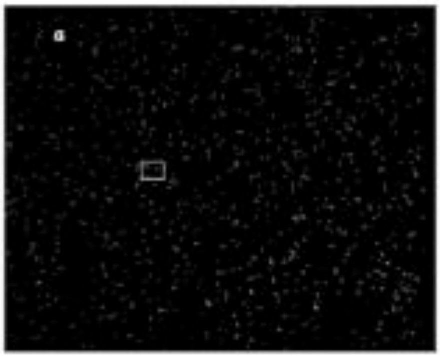
Particle Tracking Velocimetry

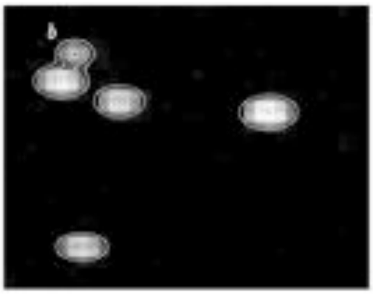
Ott, Mann, *J. Fluid Mech.*, **422**, 207, (2000)

>>Wiki software <<

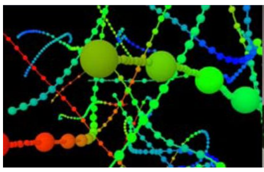


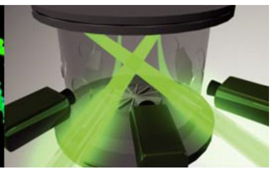




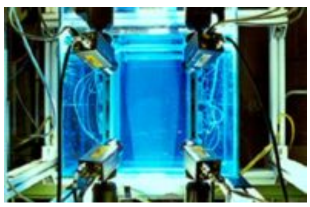


PTV tracking issues

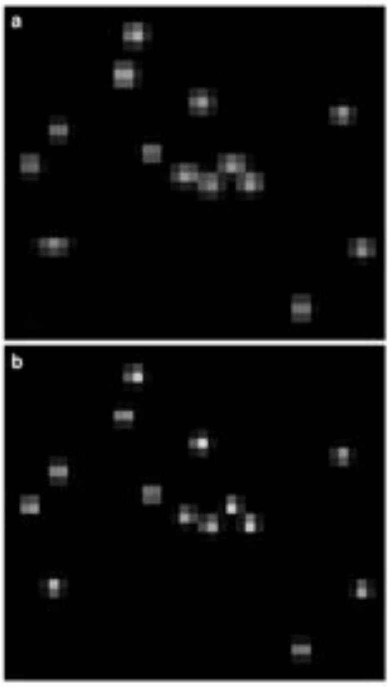


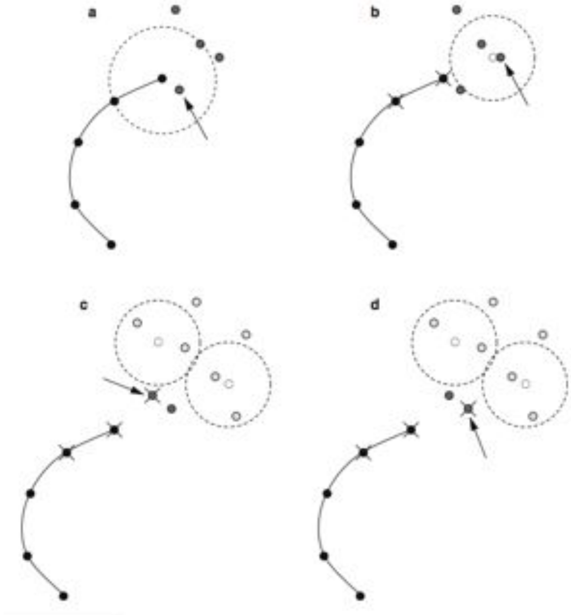


- SPEED > 25 000 fps
- REDUNDANCY >= 3 cameras
- LIGHT ≈ 200 W laser
- DATA x GB
  (…most of it useless … FPGA et al.)
- CALIBRATION perspective, …
- PROCESSING missing segments
  derivative of signal

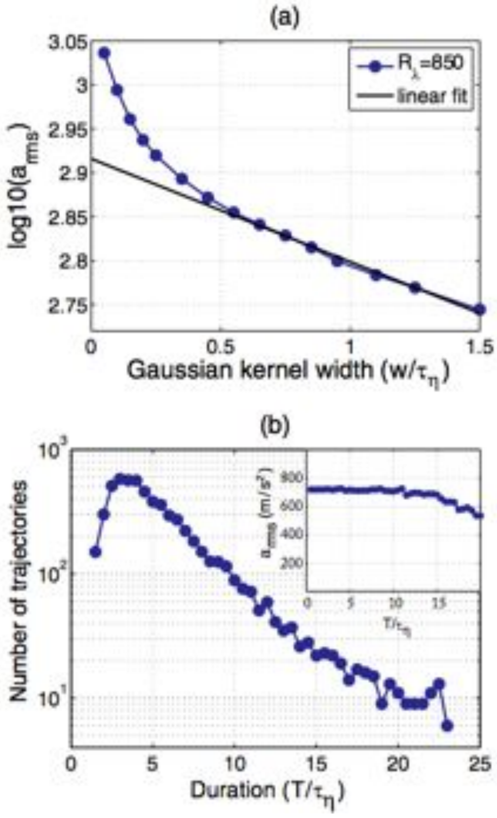


PTV tracking





PTV tracking



PTV tracking

Ayyalasomayajula et al. *PRL* **97** (2006)

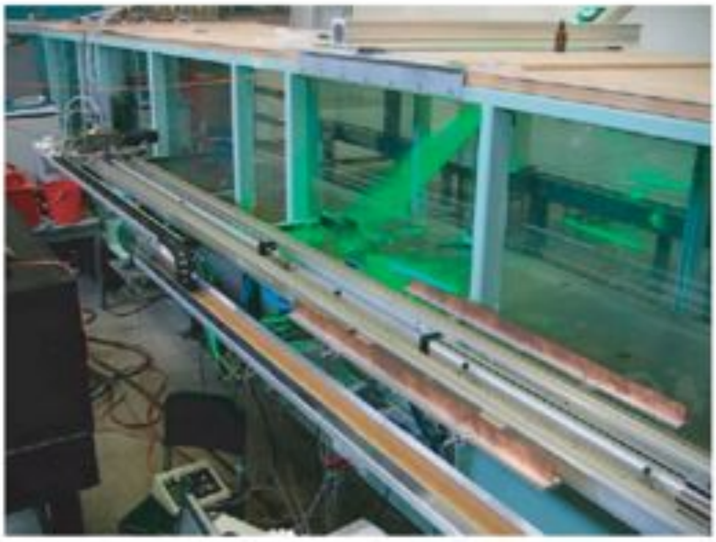


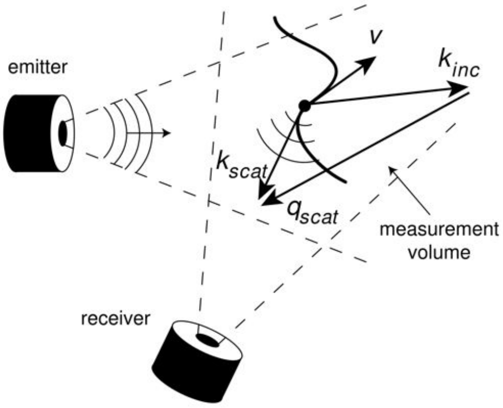
FIG. 1 (color). The wind tunnel showing the camera (far left at the beginning of its trajectory), the sled, and the laser sheet The active grid and spray system are at the tunnel entrance (just above the camera lens). The copper strips (right foreground) are the magnetic braking system for the camera sled.

PTV tracking in clouds

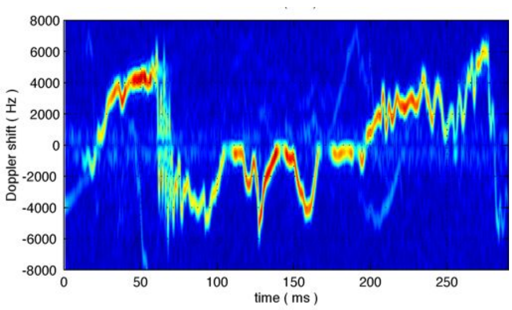
Bodenschatz Zugspitze experiment, 2010

Doppler acoustic tracking

Mordant, PhD thesis, ENS-Lyon, 2001
Mordant, Metz, JFP, Michel, *Rev Sci Inst* **76** (2005)

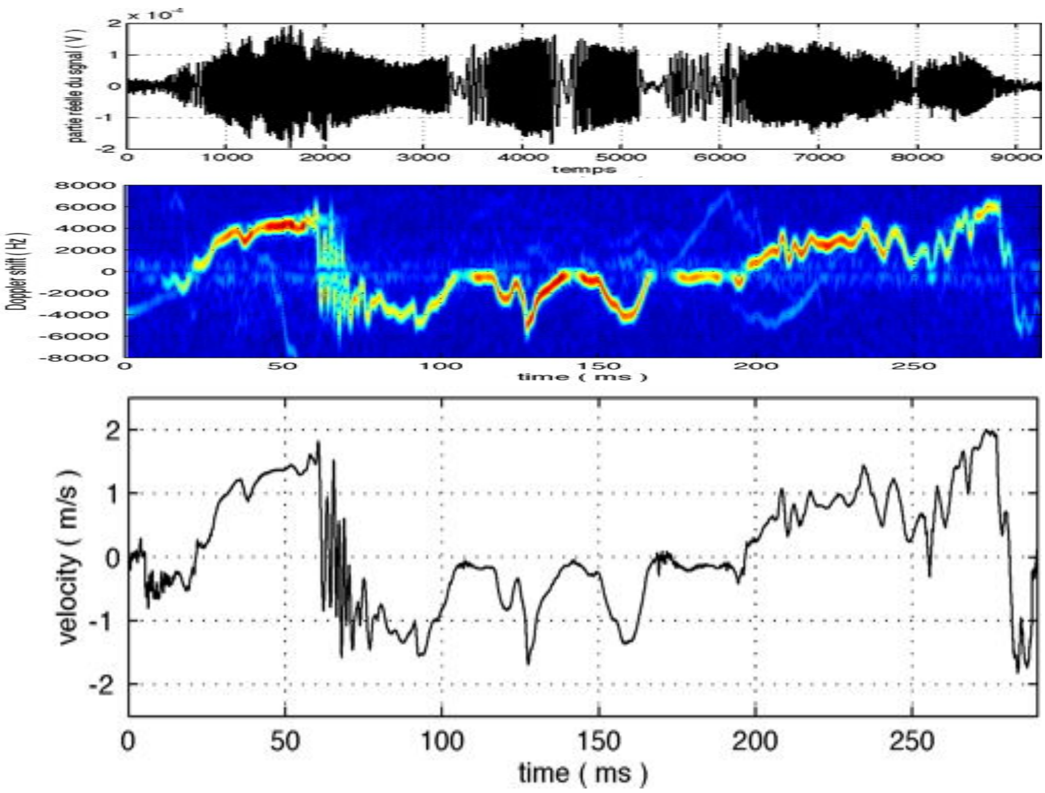


$$2\pi f(t) = \vec{q} \cdot \vec{u}(t)$$

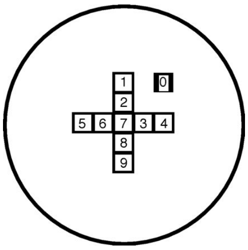


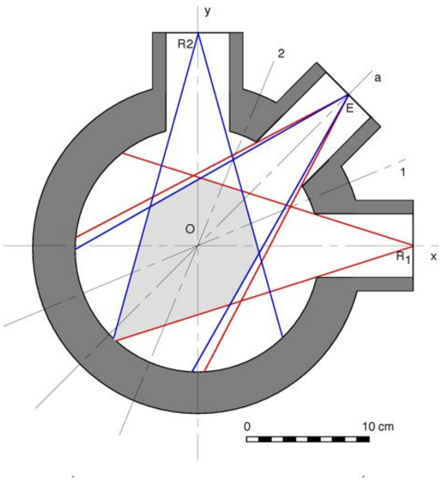
Signal processing

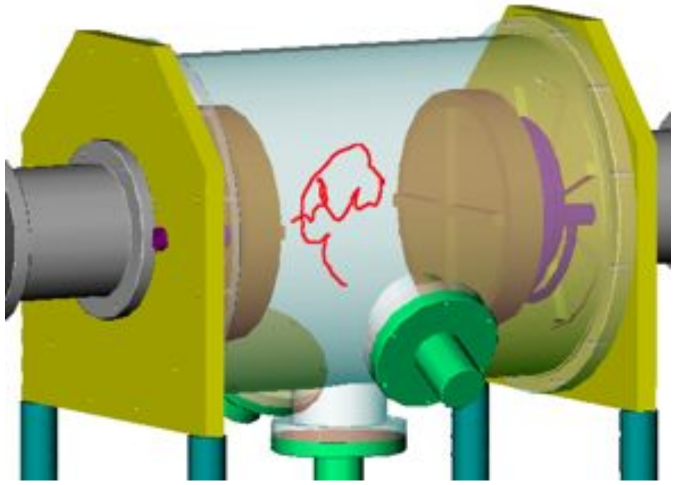
Mordant, Michel, Pinton, *J. Acoust. Soc. Am*. **112**, 108 (2002).



Measurments in VK flows

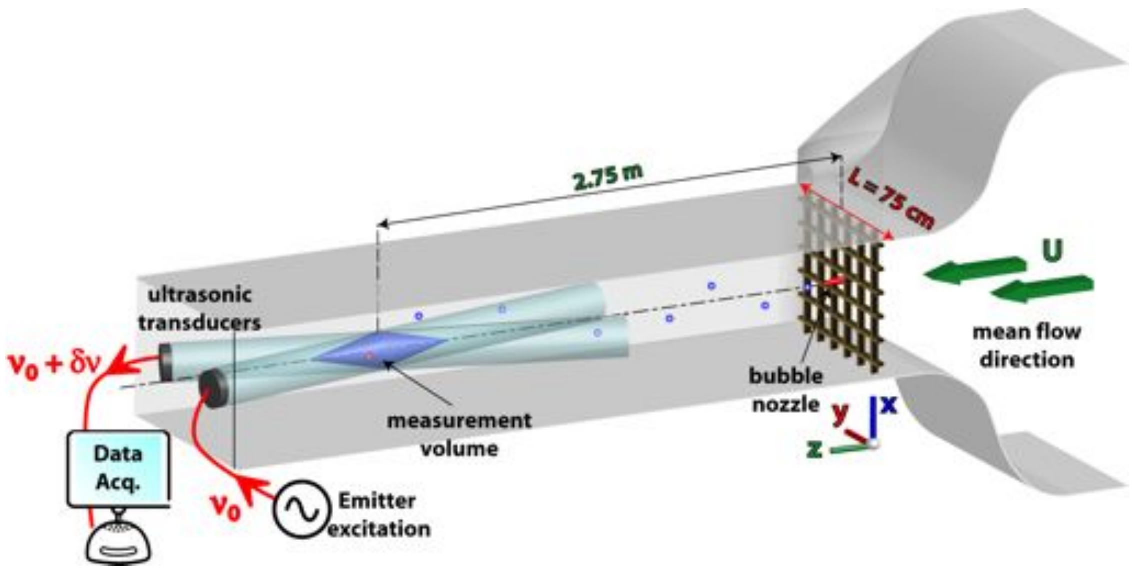






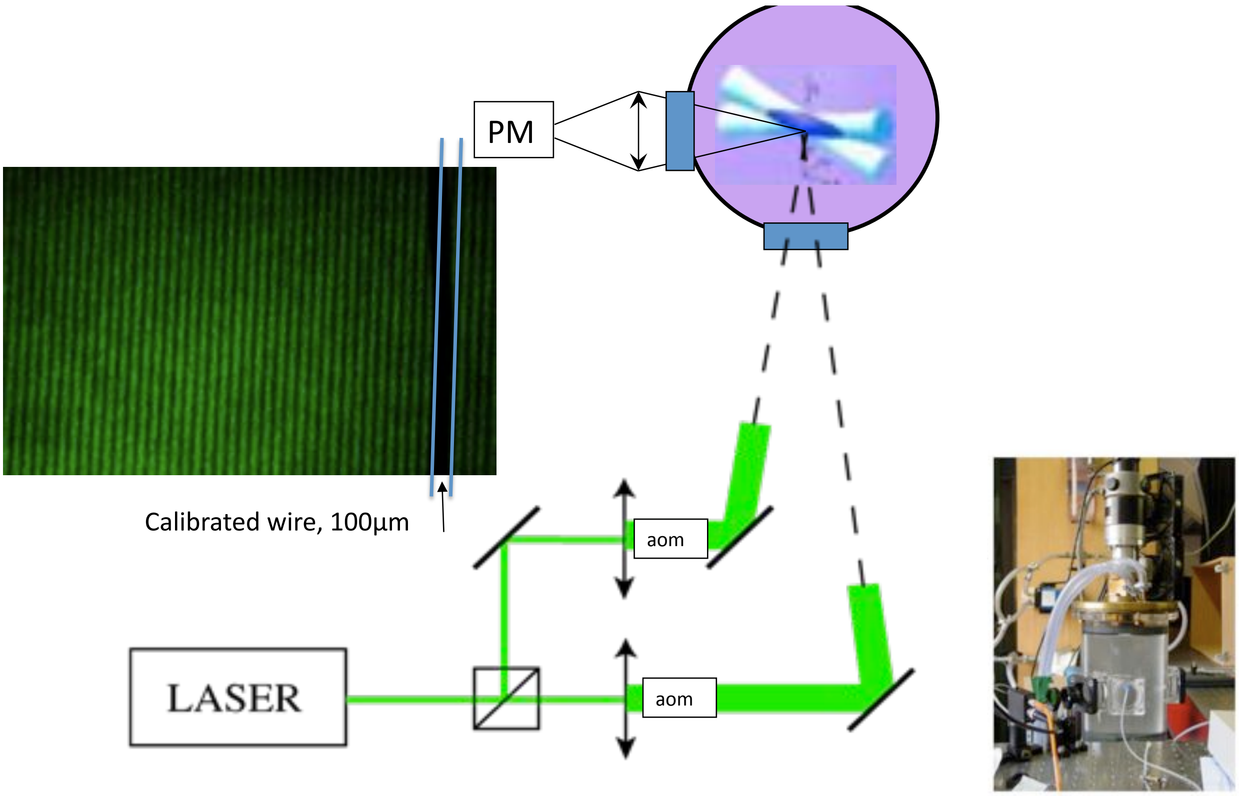
Measurements in wind tunnel @ LEGI

Qureshi, Bourgoin, Baudet, Cartellier, Gagne, *PRL* **99** (2007)
Qureshi, Arrieta, Baudet, Cartellier, Gagne, Bourgoin, *EPJ B* **97**, (2008)



Extended Laser Doppler Velocimetry

Volk, Verhille, Mordant, JFP, *EPL* **81**, 34002 (2008)



Acoustic spectroscopy

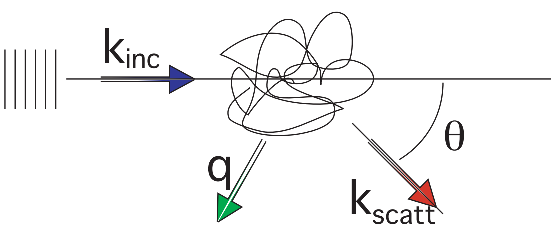
Lund & Rojas, *PhyD*, **37** (1989).

- Scattering equation, 1st Born approx

$$\partial_{tt} p_s - c^2 \Delta p_s = - c^2 \rho_0 \mathrm{div}(u_{s0} \times \Omega) + \partial_t (u.\mathrm{grad}\, p_{s0}) - p_{s0} \Delta(u.u_s)$$

- Far-field

$$p_{scatt}(R, t) \approx p_0\, g(\theta)\, M\, \Omega_{perp}(q, t)$$



quadrupole, forward
scatt amplitude -(40 to 60)dB
Doppler shift

experiment : vortex street

Baudet et al. *PRL* (1991)
Brillant et al. *EPJB* **37** (2004)

- $p_{scatt}(R, t) \approx p_0\, g(\theta)\, M\, \Omega_{perp}(q, t)$

flow: U=26 cm/s
rod: d= 3mm

b≈ 5.4 d

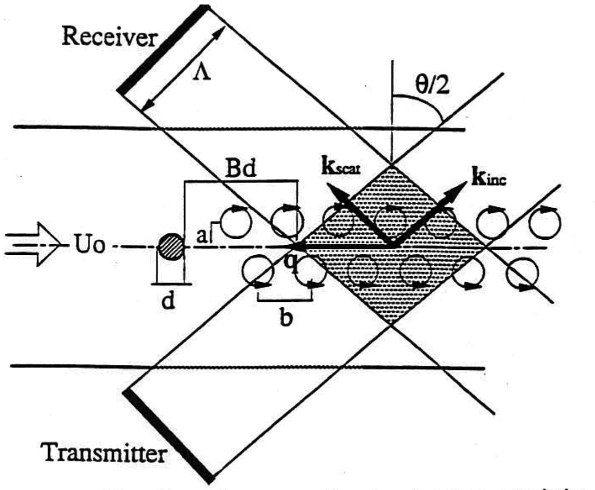


Figure 1. Experimental set-up and vortex street caracteristics.

turbulent cascade

Gervais, Baudet & Gagne, *Exp in Fluid*, **42** (2007)
Poulain et al., Flow, *Turb. and Comb.*, **72** (2004)

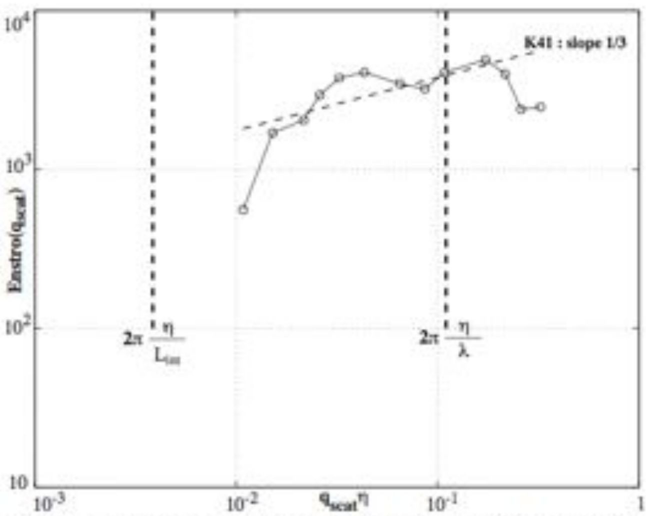
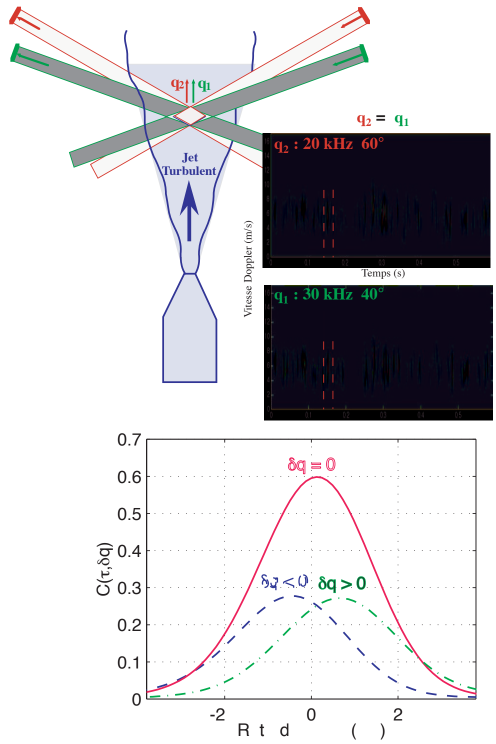


Figure 8. Spatial enstrophy spectral density as a function of the non dimensionnal wave-number in the LEGI jet ($R_\lambda \simeq 785$).



abstract

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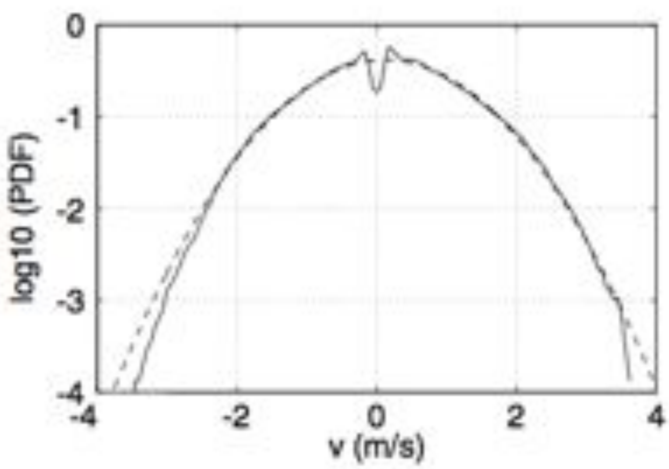
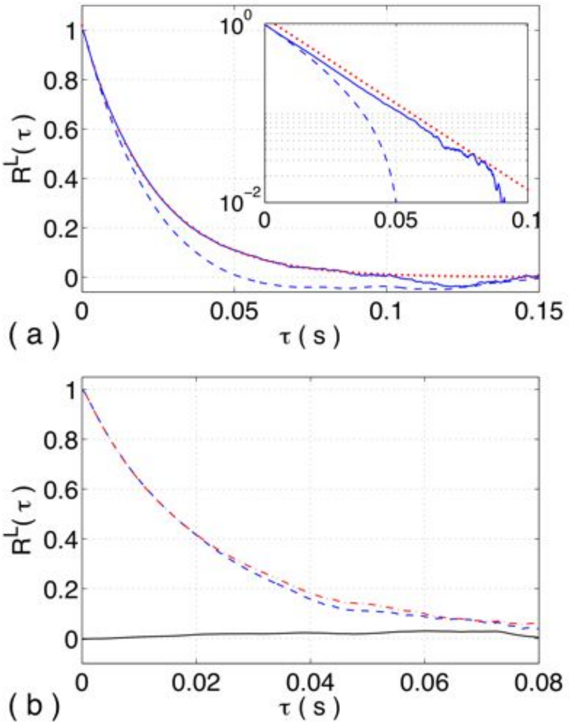
Lagrangian velocity

Mordant, et al. *NJP* **6** (2004)



velocity auto-correlation

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)

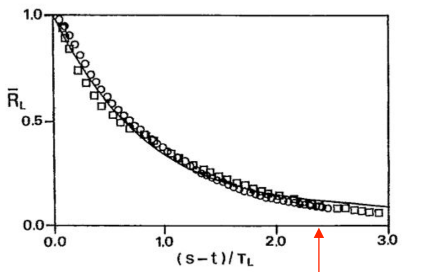


$R_L(\tau) = \langle v(t)v(t+\tau)\rangle / v^2$

$R_L(\tau) \approx \exp(-\tau / T_L)$

$T_L \approx 22$ ms (driving discs)

Valid $\tau \in [\, 10\,\tau_\eta \,, 4\, T_L \,]$



Yeung & Pope, 1989

velocity auto-correlation

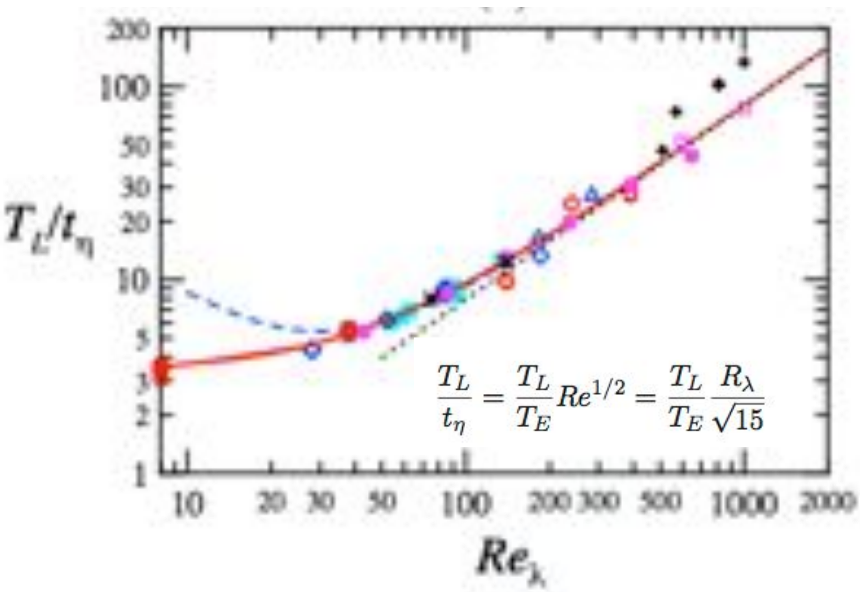
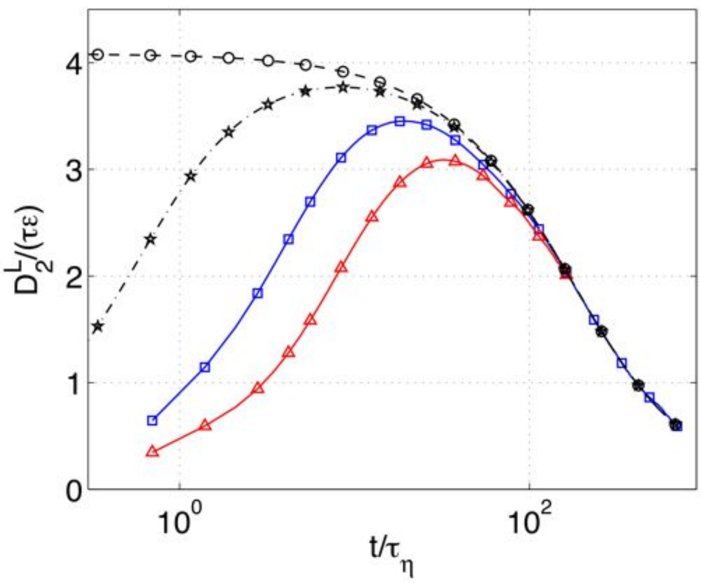


Figure 1.2 (a) Lagrangian velocity autocorrelation function, measurement from [90] at $R_\lambda = 740$ – inset: plot in semi-logarithmic scale. (b) Data for Lagrangian integral time scale. Symbols: ■, [177, 173]; •, [178]; □, Yeung (Pers. comm.); △, [19] (adjusted after pers. comm.); ⋄, [102] indirect; ∘, [30] indirect; +, [92] lab, ×, [92] DNS. Lines: – – –, second order stochastic theory Eq.(1.15); — —, empirical fit Eq. (1.16); - - -, large Re limit Eq. (1.14)

Second order structure function

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)



1mm particle 0.25mm particle
(*)Sawford 2-exp. fit (o) Exponential fit

$D^2{}_L(\tau) = < (v(t+\tau)-v(t))^2 >$

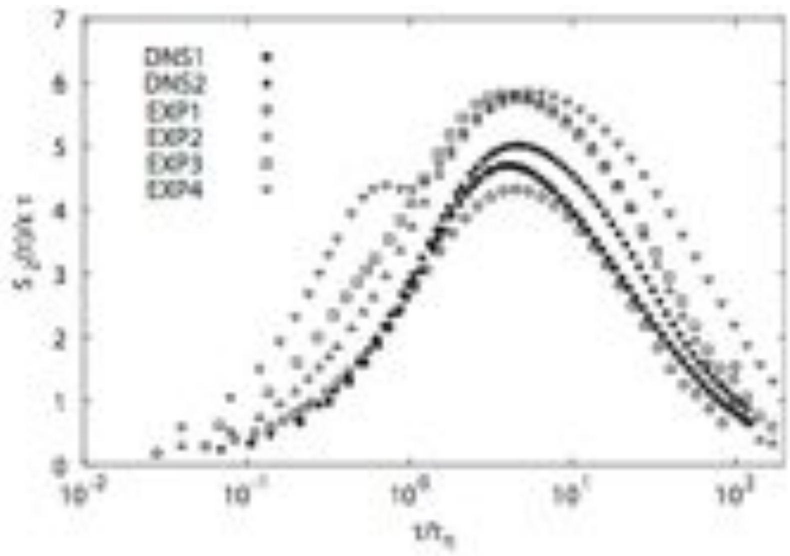
Kolmogorov 'K41'
dimensional argument
($[\varepsilon]=m^2/s^3$)

$D^2{}_L(\tau) = C_0\, \varepsilon\, \tau$

C_0 universal 'constant'
in 'inertial range'

Second order structure function

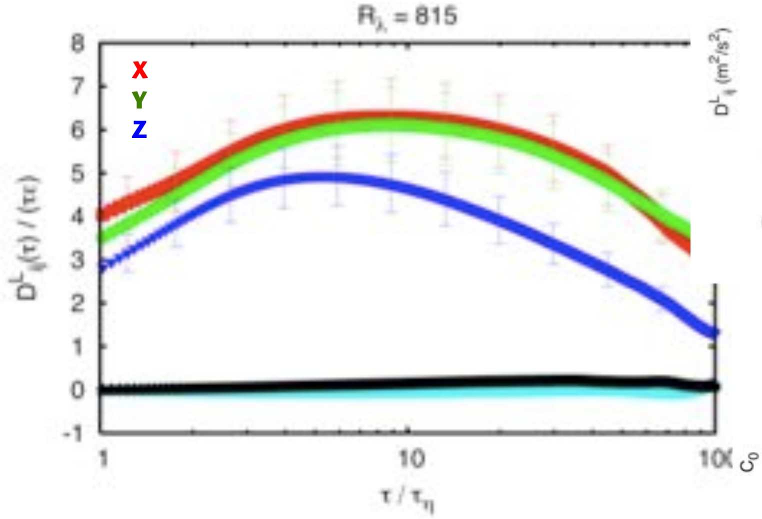
Biferale et al. PoF 20 (2008)

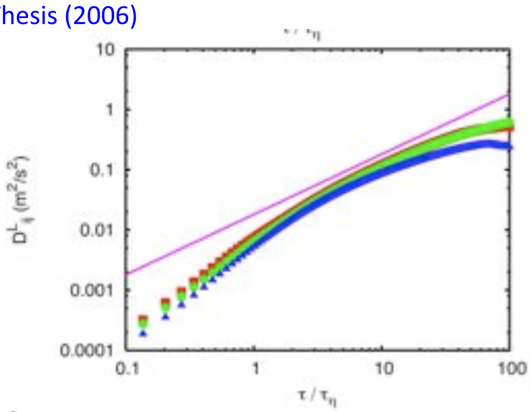


DNS R_λ = 178, 284
EXP: R_λ = 350, 690, 815

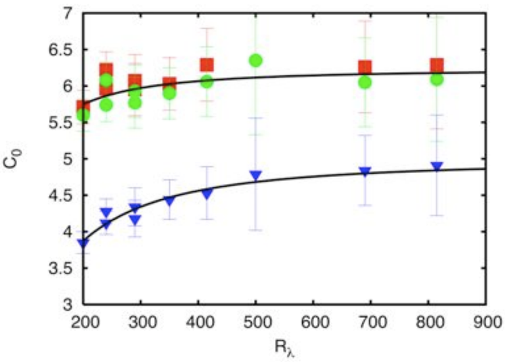
Second order structure function

Nicholas Ouellette PhD Thesis (2006)



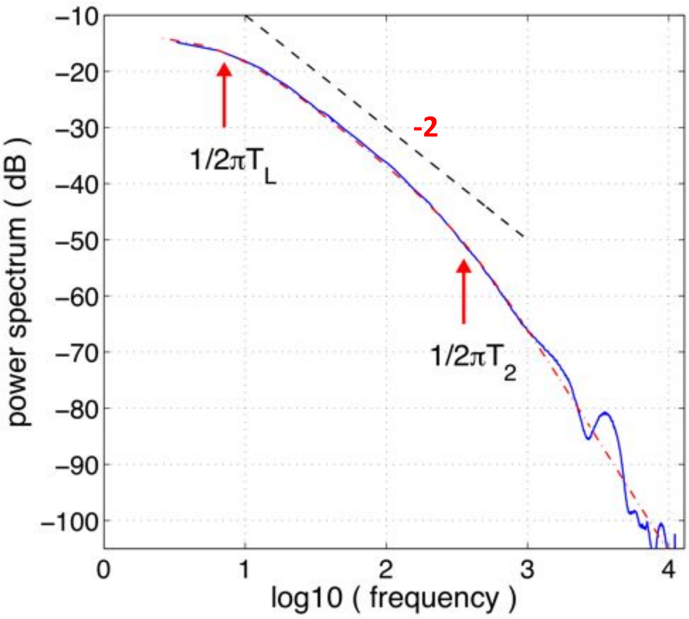


$$D_2^L(\tau) = \begin{cases} \frac{1}{3}\langle A_i^2 \rangle \tau^2 & \tau \ll t_\eta \\ C_0 \langle \epsilon \rangle \tau & t_\eta \ll \tau \ll T_L \end{cases}$$



velocity spectrum

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)



$R_L(\tau) \approx \exp(-\tau / T_L)$

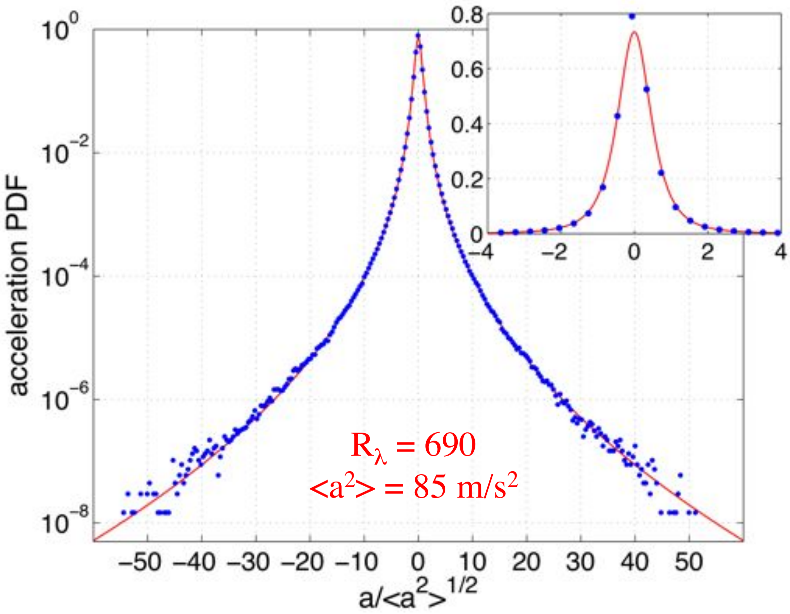
$E_L(\omega) = v^2 T_L / (1+ T_L^2 \omega^2)$

Kolmogorov

$E_L(\omega) = C_0 \varepsilon\ \omega^{-2}$

Lagrangian acceleration

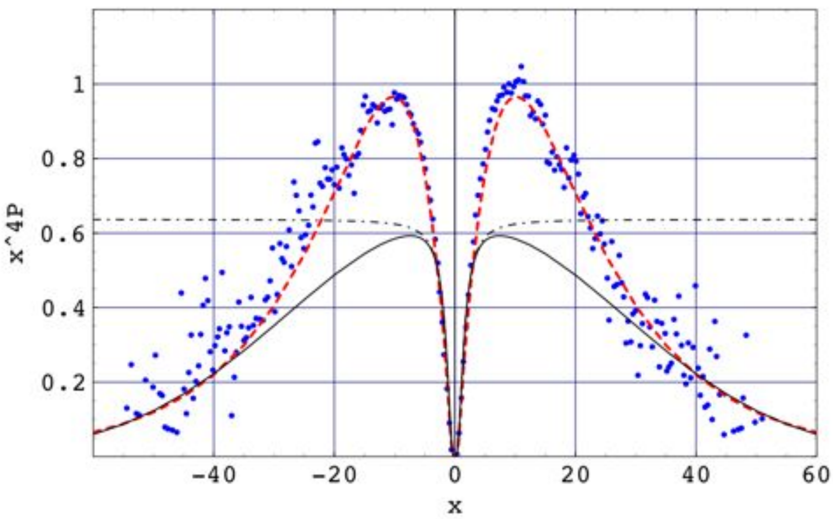
Mordant, Crawford, Bodenschatz, *Physica D*, **193**, (2004)



Heisenberg-Yaglom scaling : $< a^2 > \propto \epsilon^{3/2}\nu^{-1/2}$

Lagrangian acceleration

Mordant, Crawford, Bodenschatz, *Physica D*, **193**, (2004)



Heisenberg-Yaglom scaling : $< a^2 > \propto \epsilon^{3/2}\nu^{-1/2}$

Heisenberg-Yaglom scaling

$$< a^2 > \propto \epsilon^{3/2} \nu^{-1/2}$$

$$a_0 = 5/\left(1 + 110 R_\lambda^{-1}\right)$$

$$a_0 = 1.9 R_\lambda^{0.135}/\left(1 + 85 R_\lambda^{-0.135}\right)$$

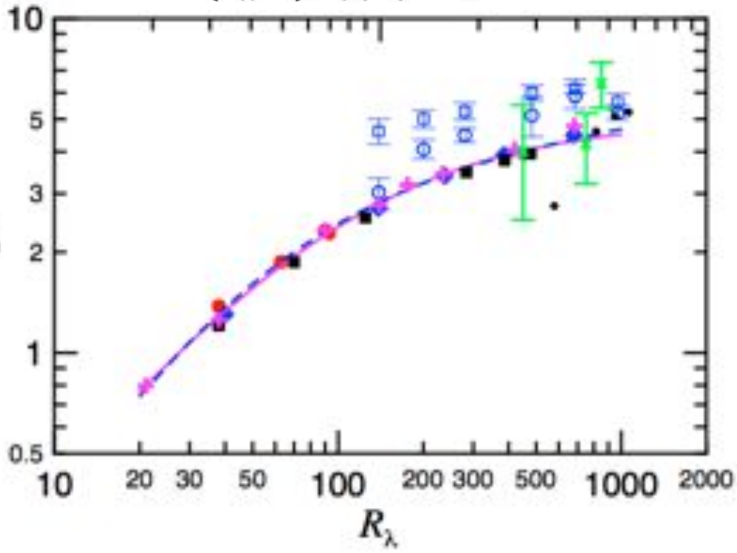
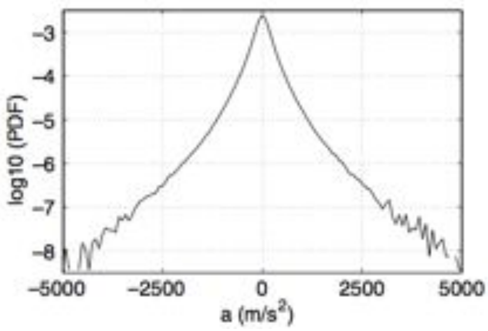


Figure 1.6 DNS and laboratory results for the non-dimensional acceleration $\frac{1}{3}\langle A_i^2\rangle / (\langle\epsilon\rangle / t_\eta) = a_0$. Symbols: •, [177]; ♦, [178] ; ■, [63]; +, [156]; ×, [161]; ○,□ [135]; •, [162]. Lines: ——, Eq. (1.22); – – –, Eq. (1.23)

lagrangian intermittency



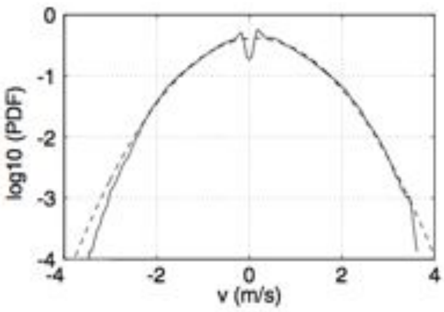
v(t+τ) – v(t)

τ ≈ t_η

τ ↗

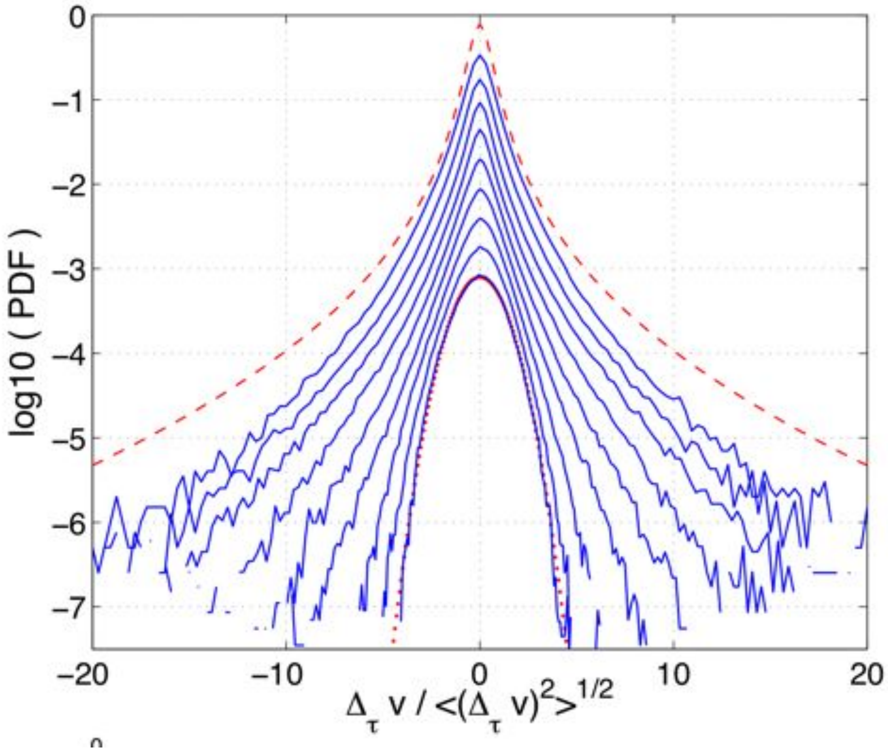
v(t+τ) – v(t)

τ ≈ T_L



lagrangian intermittency

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)



lagrangian SF exponents

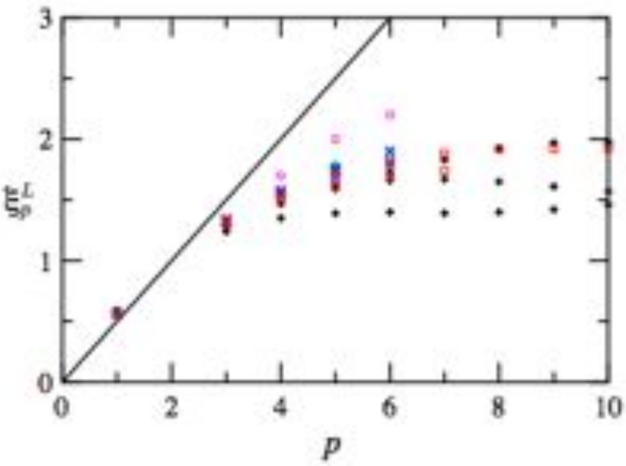
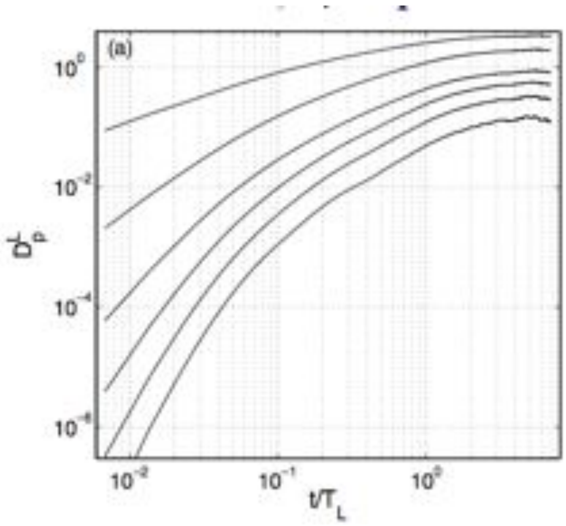
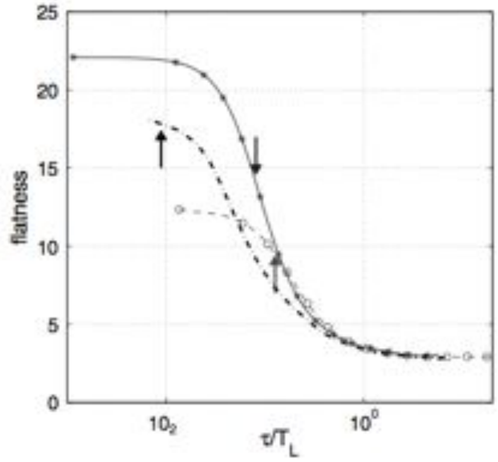


Figure 1.4 Lagrangian structure function scaling exponents from: laboratory data, [90] ▲, $R_\lambda = 740$; [92], ×, $R_\lambda = 510 - 1000$; [170], +, $R_\lambda = 200 - 815$; and DNS; [19], ○, $R_\lambda = 284$; [92], □, $R_\lambda = 75 - 140$

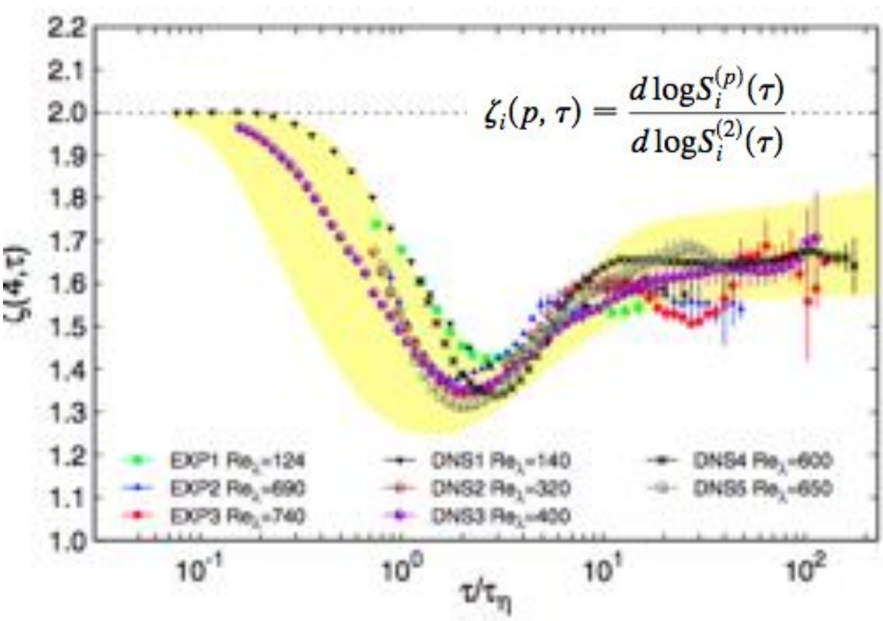
lagrangian intermittency





lagrangian SF exponents

ICTR collaboration, *Phys. Rev. Lett.*, **100** (2008)



Euler vs. Lagrange

M. Borgas. *Phil. Trans. Roy. Soc. London*, **A342**, 379, (1993)

$$\xi(q) = (1/2 + \lambda_L^2)q - \lambda_L^2 q^2/2,$$

$$D_p(\tau) \sim \langle \epsilon_\tau^{p/2} \rangle \tau^{p/2} \sim \tau^{p/2 + \alpha^L(p/2)}$$

$$S_p(\ell) \sim \langle \epsilon_\ell^{p/3} \rangle \ell^{p/3} \sim \ell^{p/3 + \alpha^E(p/3)}$$

$$\text{Richardson} \quad \ell^2 \sim \tau^3$$

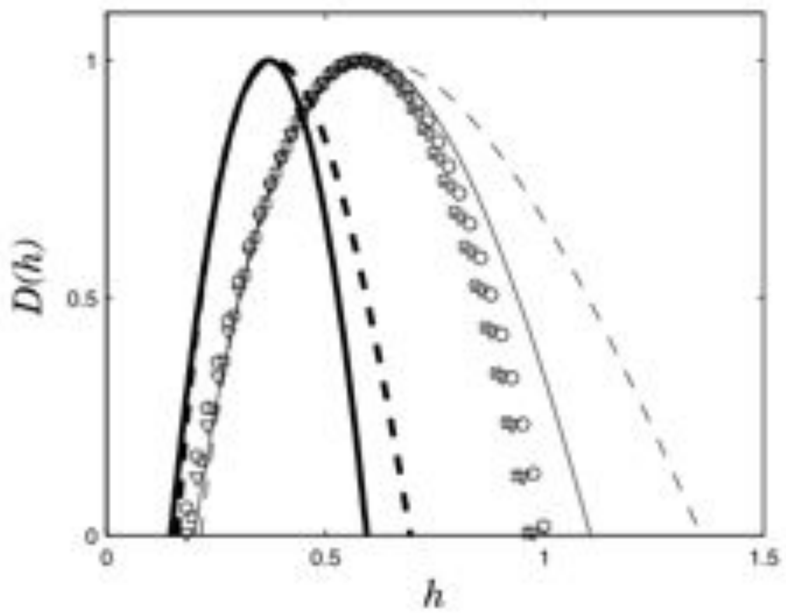
$$\lambda_L^2 / \lambda_E^2 = (3/2)^3 \simeq 3.5$$

$\lambda_E^2 \approx 0.022$ $\lambda_L^2 \approx 0.085$ $\lambda_L^2 / \lambda_E^2 \approx 3.7 \pm 0.5$

Euler vs. Lagrange

L. Chevillard et al. *PRL*, **91** (2003)
and lecture by L. Biferale

$$\mathcal{D}(h) = -h + (1+h)\mathcal{D}^E(h/(1+h)),$$



Euler vs. Lagrange

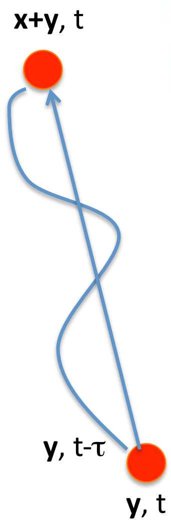
O. Kamps, R. Friedrich, R. Grauer, *PRE* **78** (2008)

$$u_e = v(\boldsymbol{y}+\boldsymbol{x},t) - v(\boldsymbol{y},t)$$

$$u_l = v(\boldsymbol{y}+\tilde{\boldsymbol{x}}(\boldsymbol{y},\tau,t),t) - v(\boldsymbol{y},t-\tau)$$

$$u_{el} = v(\boldsymbol{y}+\tilde{\boldsymbol{x}}(\boldsymbol{y},\tau,t),t) - v(\boldsymbol{y},t)$$

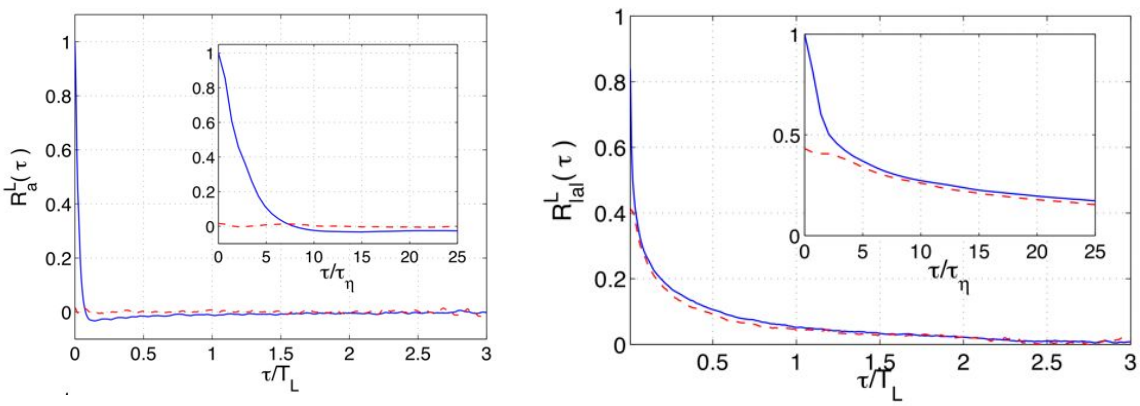
$$u_p = v(\boldsymbol{y},t) - v(\boldsymbol{y},t-\tau)$$

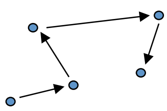


$$f_l(v_l;\boldsymbol{y},\tau,t) = \left\langle \int dv_e\ \hat{f}_p(v_l - v_e;\boldsymbol{y},\tau,t)\hat{f}_{el}(v_e;\boldsymbol{y},\tau,t)\right\rangle = \int dv_e\ f_p(v_l - v_e|v_e;\boldsymbol{y},\tau,t) f_{el}(v_e;\boldsymbol{y},\tau,t).$$

Lagrangian acceleration

Mordant, Delour, Leveque, Michel, Arnéodo, Pinton, *J. Stat. Phys.*, **113**, 701, (2002)



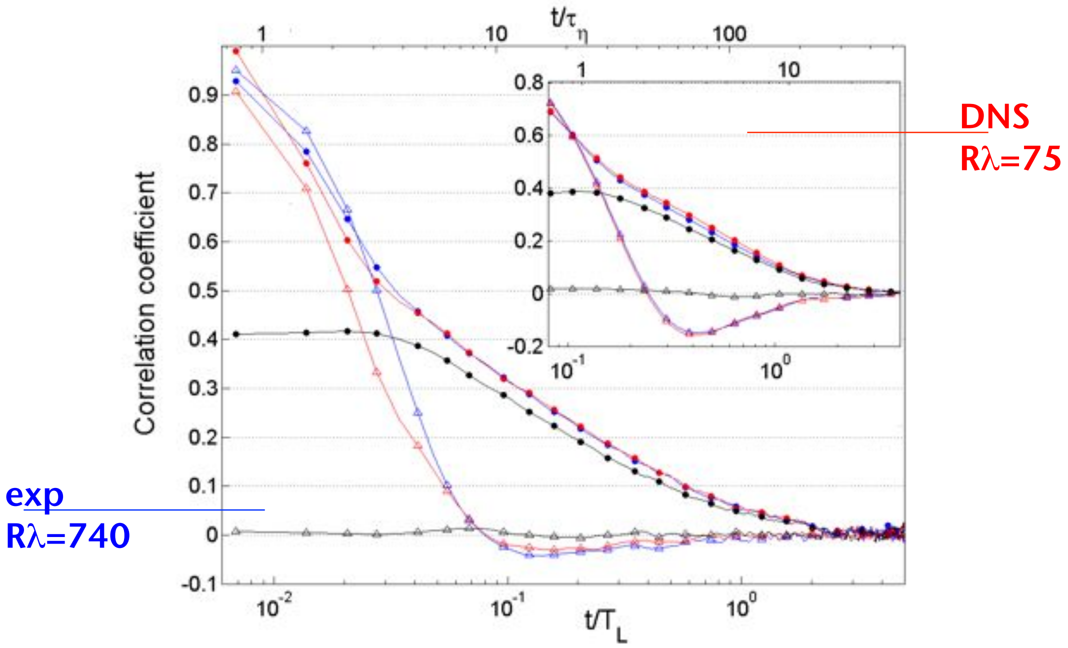


Short-time correlation for the acceleration direction

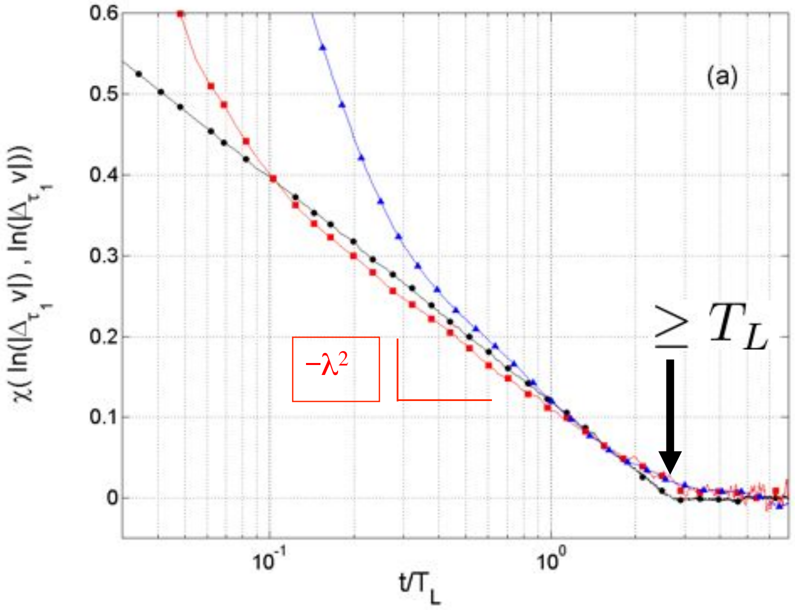
Long-time correaltion for the acceleration magnitude

Correlation of velocity increments

- $\Delta u_{\tau 0}(t) = v(t+\tau_0)-v(t)$ -> $C(t) = < u_{\tau 0}(t')\, u_{\tau 0}(t'+t) >_{t'}$



A Lagrangian Random Walk



$< \log| u_{\tau 0}(t')| - < \log| u_{\tau 0}| >)(\log| u_{\tau 0}(t'+t)| - < \log| u_{\tau 0}| >_{t'} \propto -\lambda^2 \log(t)$

MRW model

Bacry, Delour, Muzy, *Phys. Rev. E*, **64**, (2001).

stochastic equation for the velocity increments

$$d_t u = -\gamma(u)u + \xi(t)$$

'K41' theory : $\xi(t)$ is δ-correlated noise,

Model, from observations :

$$\xi(t) = e^{\omega(t)} G(t)$$

G(t) : gaussian , white in time, and :

$$\langle \omega(t)\omega(t+\Delta t)\rangle_t = -\lambda^2 \log(\Delta t / T_L)$$

Langevin models of Lagrangian acceleration

Aringazin & Mazhitov, *Phys. Rev. E*, **68**, 026305, (2004)

$$d_t a = \gamma F(a) + \sigma L(t) \quad \beta = \gamma/\sigma^2$$

$$P(a) = \int_0^\infty d\beta P(a|\beta) f(\beta)$$

when L(t) is δ-correlated Gaussian white noise

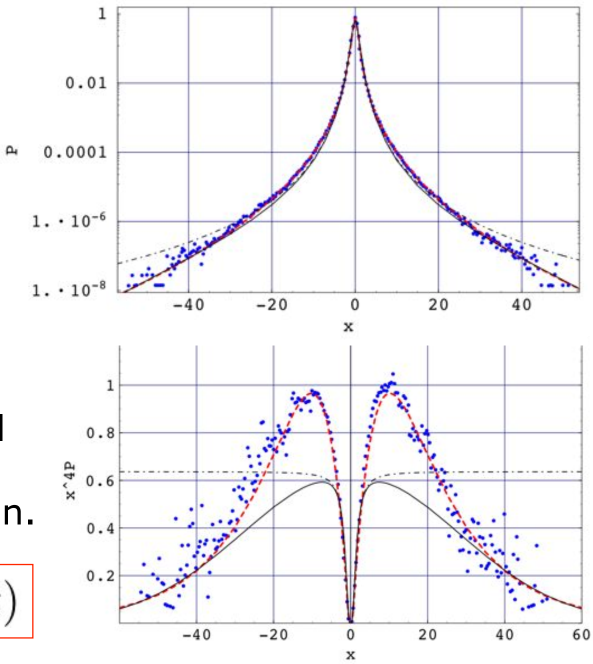
F(a)=-a

$$P(a|\beta) = C(\beta) \exp[-\beta a^2/2]$$

QUESTION : statistics $f(\beta)$?

Langevin models of Lagrangian acceleration

Aringazin & Mazhitov, *Phys. Rev. E*, **68**, 026305, (2004)



- f(β) : χ-square distribution
- f(β) : log-normal distribution
- unifying concept : β = β(u)
  - β(u) = u^2 : χ^2
  - β(u) = exp(u) : log-normal
    - associated Langevin eqn.

$$\partial_t a = \gamma F(a) + e^{\omega} L(t)$$

    - link with Laval-Dubrulle-Nazarenko turbulence model

summary 1

1. Time and space resolution is available.

2. K41 theory = random walk in velocity space, from Lagrangian viewpoint.

3. Corrections (very important in terms of forces) correspond do memory effects.
   Integrating the statistics of force from dissipative to integral scale is more acurate than asymptotic inertial range theories. As usual, the difficulty lies in the treatment of pressure.

abstract

- Lecture 1:

  - Review of experimental techniques for lagrangian measurements,

  - The dynamics of tracers, single particle statistics.

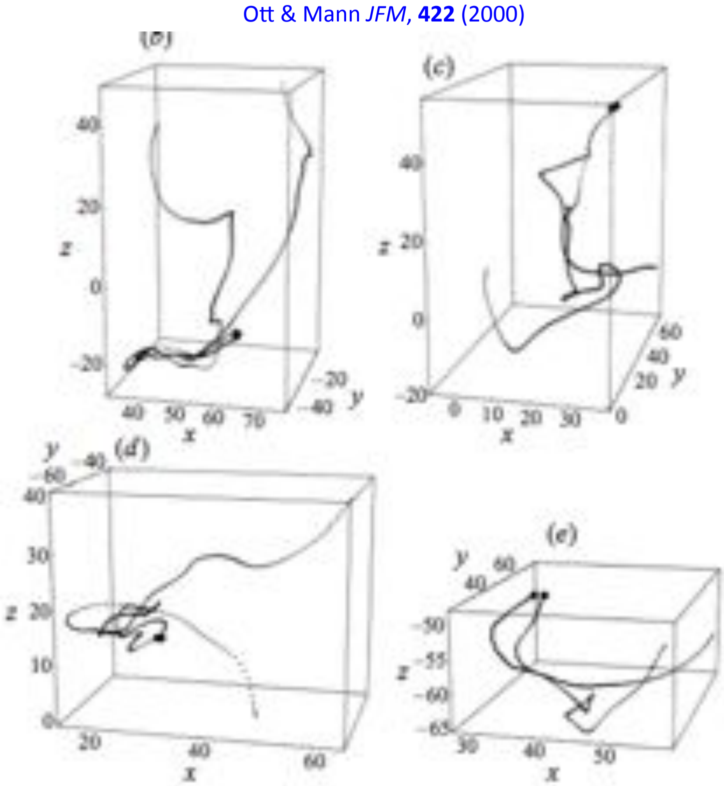
- Lecture 2:

  - Issues and results associated with multiparticle statistics,

  - The dynamics of inertial particles: size effects and density effects.

pair dispersion

Ott & Mann *JFM*, **422** (2000)



pair dispersion

Richardson –Obukhov law : (RO) $\langle r^{+2}(t) \rangle = g \langle \epsilon \rangle t^3$

Batchelor small time limit : (B) $\langle r^{+2}(t) \rangle - r_0^2 = \langle (\Delta u(r_0))^2 \rangle t^2$

$$= \tfrac{11}{3} C \langle \epsilon \rangle^{2/3} r_0^{2/3} t^2$$

(RO) for $t >> t_0$, (B) for $t << t_0$ with $t_0 \approx \varepsilon^{-1/3} r_0^{2/3}$

no intermittency corrections expected for (RO)
possible correction for (B)

pair dispersion

very small time limit : $\left\langle r^{+2}(t) \right\rangle = r_0^2 \exp(\gamma t / t_\eta)$

for $t_0 << t_\eta$, $r_0 << \eta$: exponential separation $t/t_\eta = \ln(\eta^2/r_0^2)/\gamma$

the particules never separate !

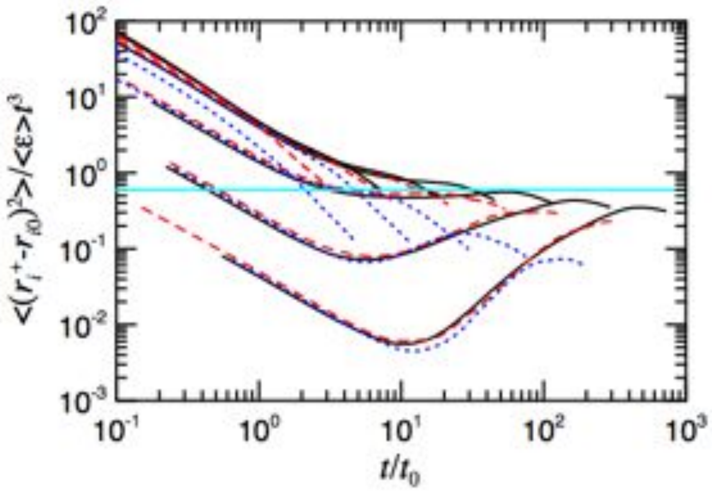
very large time limit : $\left\langle r^{+2}(t) \right\rangle = 2\sigma_x^2 \delta_{ii} = 12 \sigma_u^2 T_L t \qquad (t \gg T_L)$

independent motions

NB : r_0, t_0, t_η, T_L, …. many many scales !
for a limited Lagrangian scale separation.

pair dispersion

Sawford et al. *Phys. Fluids*, **20** (2006)



$$\left\langle r^{+^2}(t)\right\rangle = g\langle\epsilon\rangle t^3$$

$0.5 < g < 1.2$

Figure 1.9 Relative dispersion plots in inertial sub-range scaling using the length and time scales r_0 and t_0. Initial separations are nominally, from bottom to top, $r_0/\eta = 1/4, 1, 4, 16, 64,$ and 256. Lines: - - -, $R_\lambda = 38$; – – –, $R_\lambda = 240$; — —, $R_\lambda = 650$. The horizontal line is at a value of 0.6

pair dispersion

Bourgoin et al. *Science*, **311** (2006)

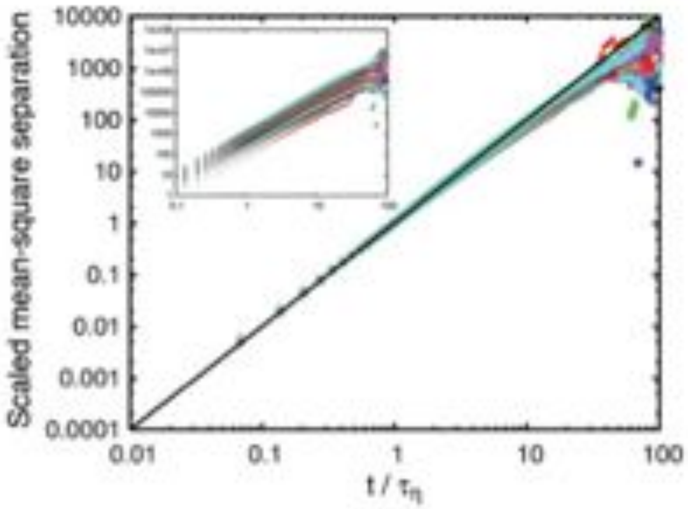


Fig. 2. Evolution of the mean square particle separation. The mean square separation between particle pairs is plotted against time for 50 different initial separations at a turbulence level of R_λ = 815, with the time axis normalized by the Kolmogorov scales. Each curve represents a bin of initial separations 1 mm wide (≈43η), ranging from 0 to 1 mm to 49 to 50 mm. The curves are scaled by the constant $(\frac{11}{3})C_2(\varepsilon\Delta_0)^{2/3}$ (Eq. 1). The data collapse onto a single universal power law. The bold black line is the power law predicted by Batchelor (11). Because the smallest Δ_0 measured is not in the inertial range, we do not expect it to scale perfectly as t^2, and indeed it does not scale as well as the larger Δ_0. The inset shows the same curves scaled simply by the Kolmogorov length, for which we see no scale collapse. For both plots, we see no Richardson-Obukhov t^3 scaling.

pair dispersion

Berg et al. *PRE*, **74** (2006)

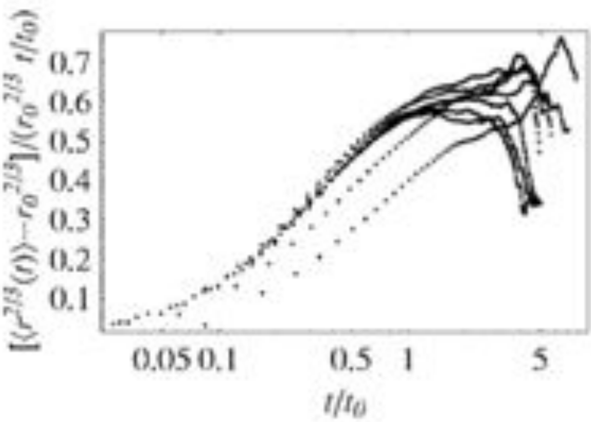
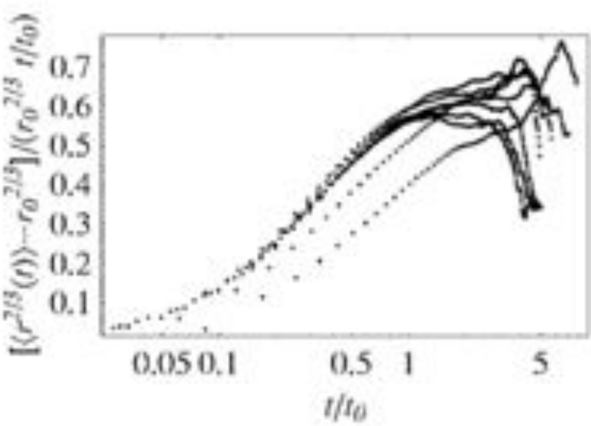


FIG. 5. $(\langle r^{2/3}\rangle - r_0^{2/3})/(r_0^{2/3} t/t_0)$ vs t/t_0. The different curves correspond to the different bins.

pair dispersion

Berg et al. *PRE*, **74** (2006)

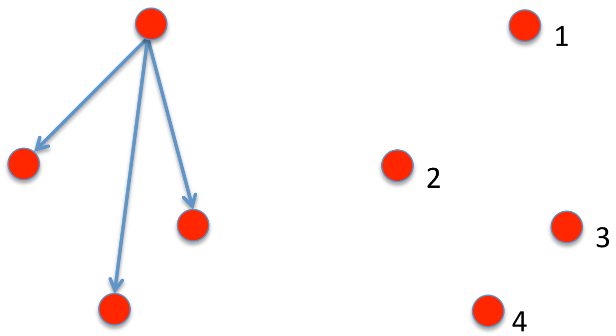


Again, a continuous description from dissipative to integral scale would be more efficient than asymptotic theories.

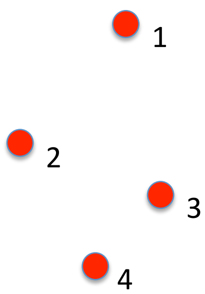
N > 2 : the geometry of turbulence

Non linear Navier Stokes term : u.grad(u) : triadic interactions

$\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$



tetrads



Define the 3 vectors :

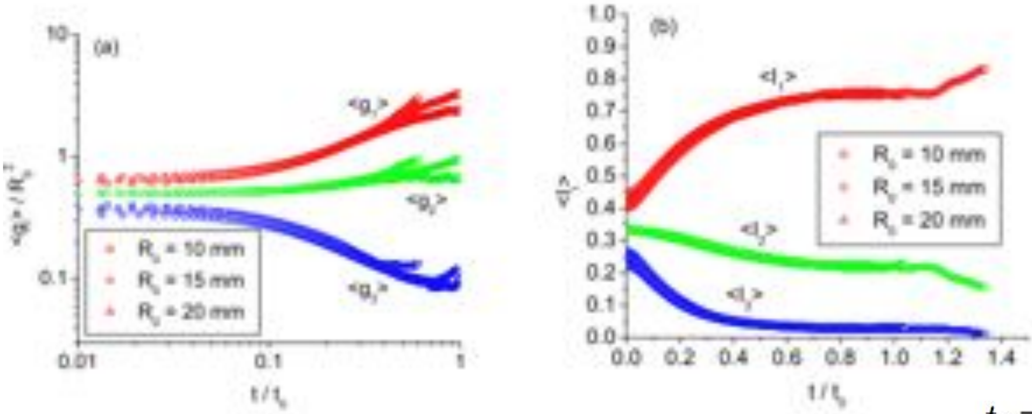
$$\boldsymbol{\rho}^{+(1)} = \frac{1}{\sqrt{2}}\left(\boldsymbol{x}^{+(2)} - \boldsymbol{x}^{+(1)}\right)$$
$$\boldsymbol{\rho}^{+(2)} = \frac{1}{\sqrt{6}}\left(2\boldsymbol{x}^{+(3)} - \boldsymbol{x}^{+(2)} - \boldsymbol{x}^{+(1)}\right)$$
$$\boldsymbol{\rho}^{+(3)} = \frac{1}{\sqrt{12}}\left(3\boldsymbol{x}^{+(4)} - \boldsymbol{x}^{+(3)} - \boldsymbol{x}^{+(2)} - \boldsymbol{x}^{+(1)}\right)$$

build the matrix M = $\rho_{ij}\rho_{ij'}$
with eigenvalues $g_1 > g_2 > g_3$, then :

tetrahedron size : $R^2 = g_1 + g_2 + g_3$

tetrads

Xu et al. *NJP*, **10** (2008)



$$t_0 \equiv (R_0^2/\epsilon)^{1/3}$$

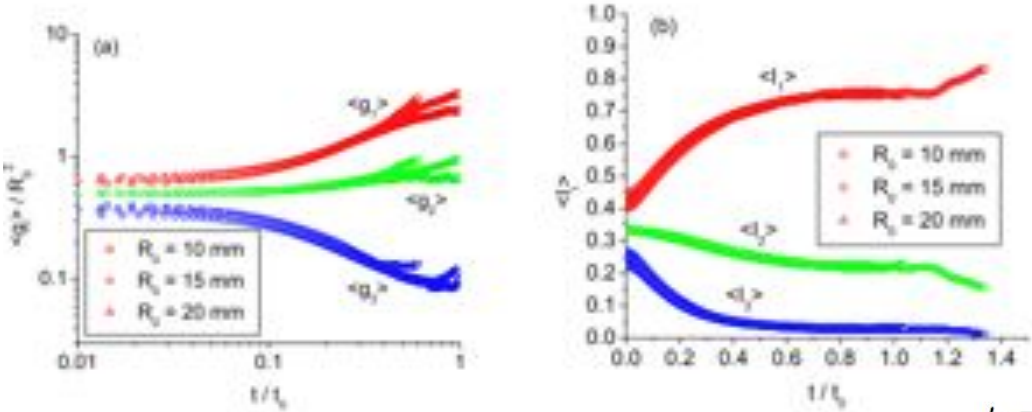
tetrahedron shape : $I_i = g_i/R^2$

$I_1 \approx I_2 >> I_3$: pancake

$I_1 >> I_2 \approx I_3$: needle

tetrads

Xu et al. *NJP*, **10** (2008)



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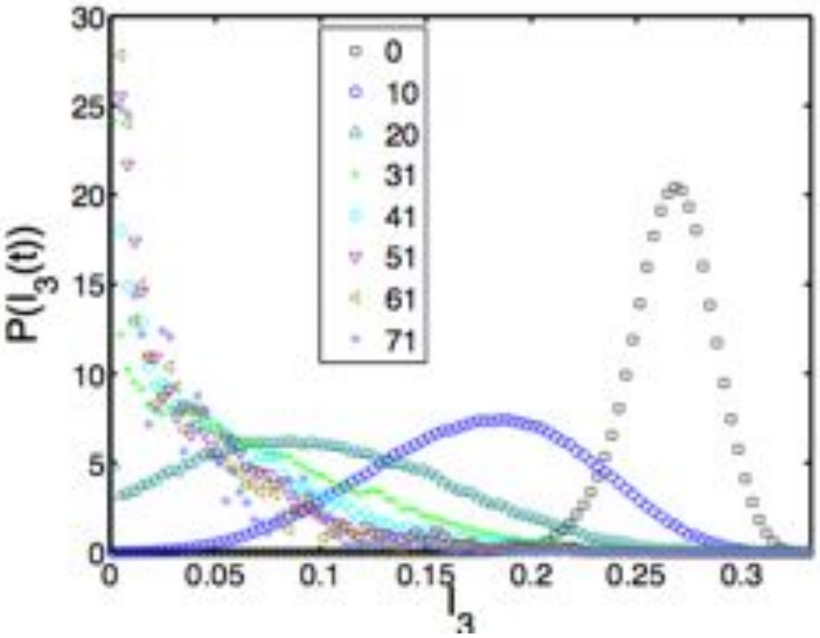
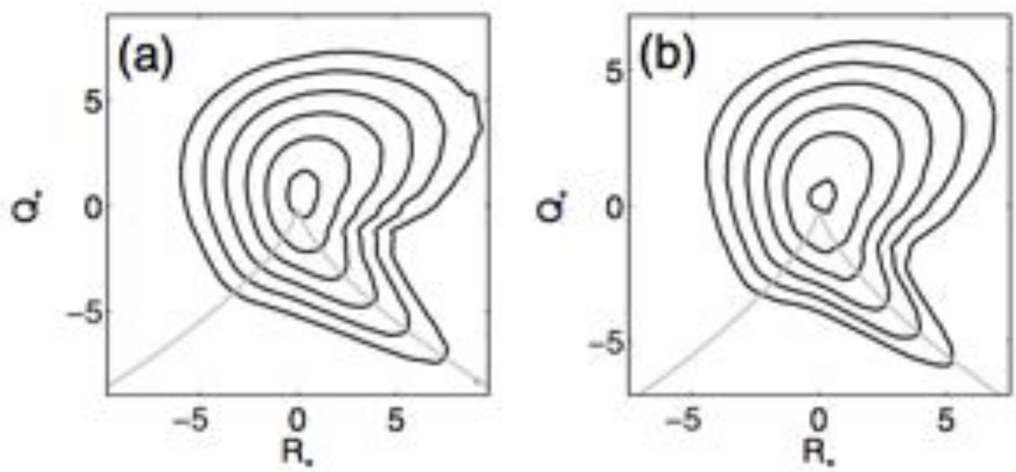
tetrads

Xu et al. *NJP*, **10** (2008)



more on tetrads …

Chertkov, Pumir, Shrainman, *PoF*, **11** (1999)
Pumir, Naso, *NJP*, **11** (2010)

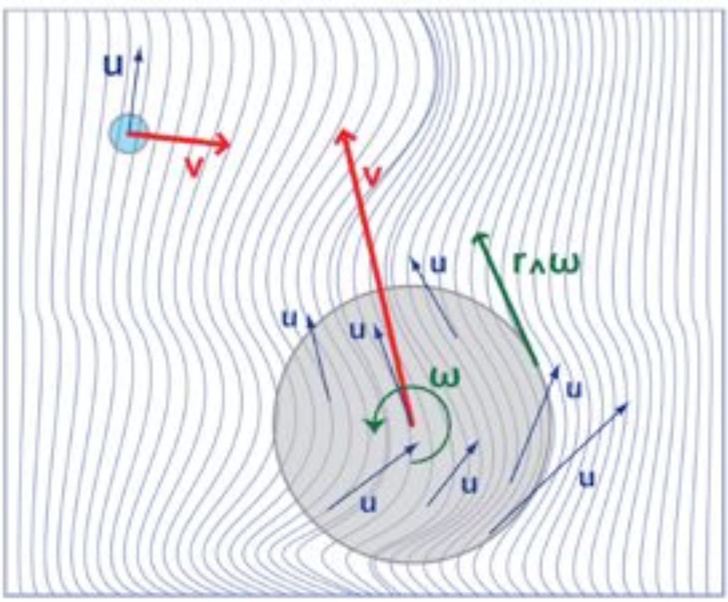


abstract

- Lecture 1:
  - Review of experimental techniques for lagrangian measurements,
  - The dynamics of tracers, single particle statistics.

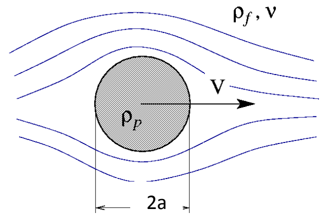
- Lecture 2:
  - Issues and results associated with multiparticle statistics,
  - The dynamics of inertial particles: size effects and density effects.

Motion, with D≠0



Forces, at $Re_p \to 0$

$$
\begin{aligned}
\rho_p \frac{d\mathbf{v}}{dt} = \rho_f \frac{D\mathbf{u}}{Dt} + (\rho_p - \rho_f)\mathbf{g} \quad & \text{gradient pressure, buoyancy} \\
- \frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right) \quad & \text{viscous drag} \\
- \frac{\rho_f}{2}\left(\frac{d\mathbf{v}}{dt} - \frac{D}{Dt}\left[\mathbf{u} + \frac{a^2}{10}\nabla^2\mathbf{u}\right]\right) \quad & \text{added mass} \\
- \frac{9\rho_f}{2a}\sqrt{\frac{\nu}{\pi}}\int_0^t \frac{1}{\sqrt{t-\zeta}}\frac{d}{d\zeta}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right) d\zeta \quad & \text{history} \\
- C_L \rho_f (\mathbf{v} - \mathbf{u}) \times \boldsymbol{\omega} \quad & \text{lift}
\end{aligned}
$$



Maxey, M. & Riley, J. 1983 Equation of motion of a small rigid sphere in a nonuniform flow. Phys. Fluids **26**, 883-889,
Gatignol, R. 1983 J. Mec. Theoric. Appl. 1, 143, and T. Auton, J. Fluid Mech. 183, 199 (1987), T. Auton et al., J. Fluid Mech. 197, 241 (1988)

Forces

Identification of contributions:

-**Drag**:

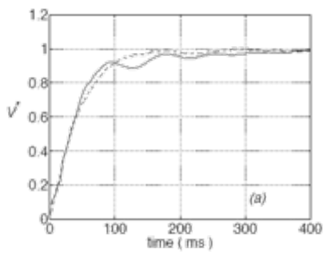
« easy » but dynamics ≠ quasistatic

- **History** :

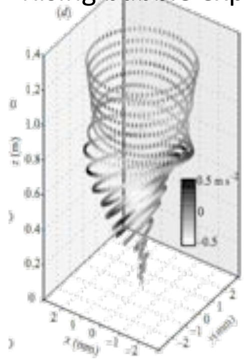
Thomas, *PoF* 1992;
Mei *JFM* 1992
Mordant et al. *EPJB* 2000

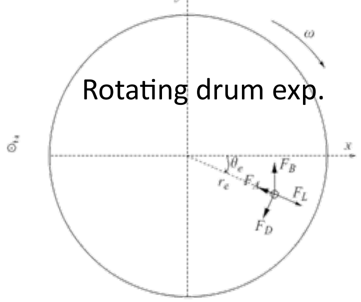
- **Lift** :

Magnaudet & Eames *Ann. Rev.* 2000;
Shew et al. *JFM* 2006, *PRL* 2006
van Nierop et al. *JFM* 2007
Rastello et al. *JFM* 2009



Rising bubble exp.

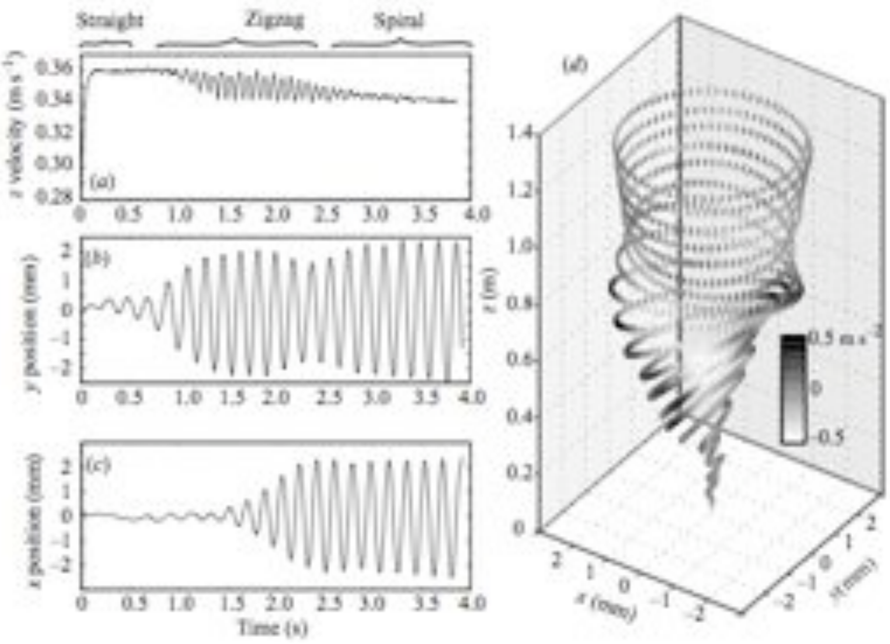




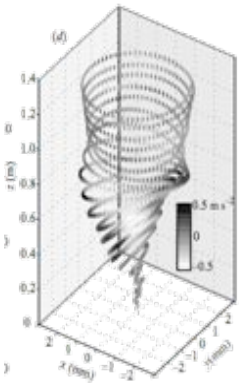
In a turbulent flow ?

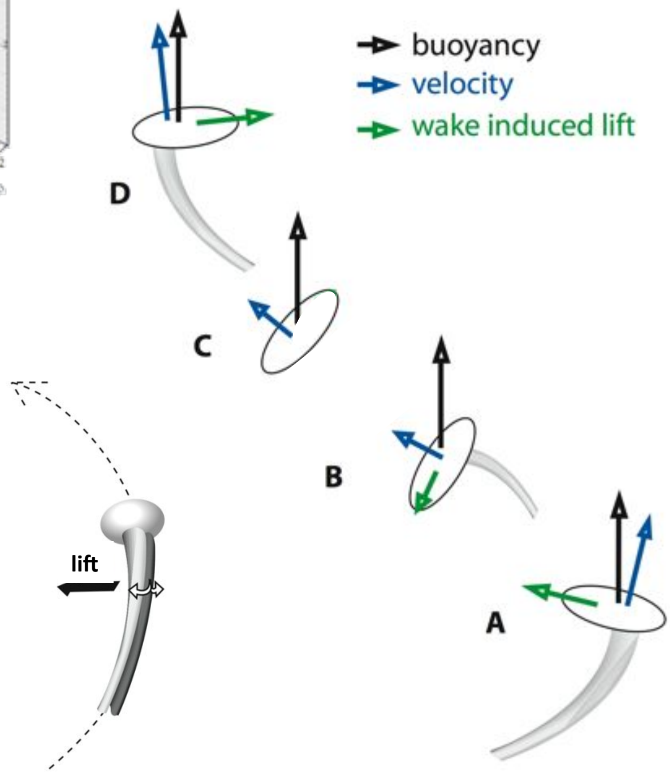
Bubbles

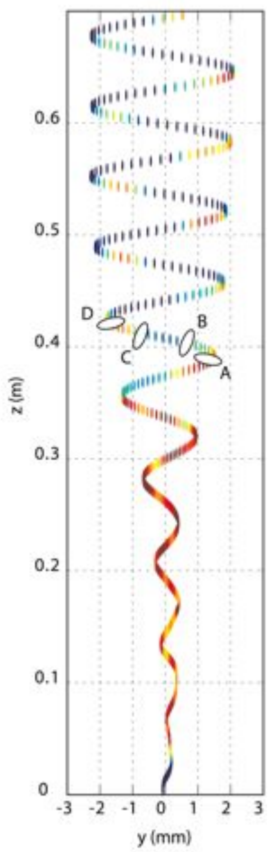
Mordant, Pinton *Eur. J. Phys. B*, **18**, (2000)



Bubbles

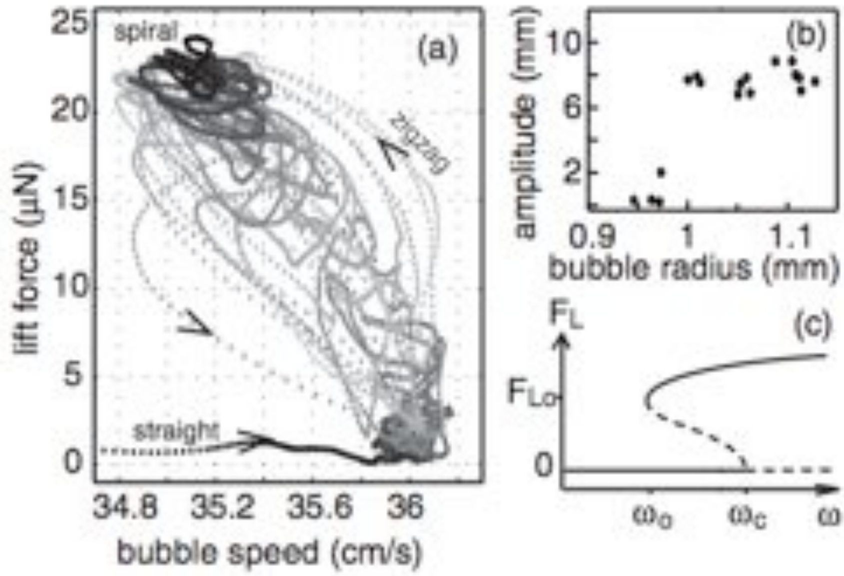






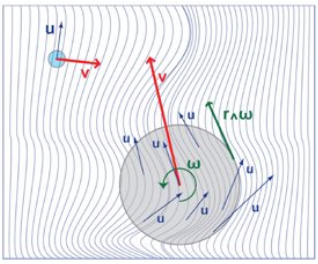
Bubbles

Shew, JFP, *PRL* 97, (2006)



$$\frac{dF_L}{dt} = \frac{1}{\tau}\left(\frac{U\chi^{5/3}/R - \omega_c}{\omega_c} F_L + k' \frac{F_L^3}{F_{L0}^2} - k' \frac{F_L^5}{F_{L0}^4}\right)$$

dynamical questions



- acceleration (force) variance
- response time
- PDF of fluctuations

- orientations

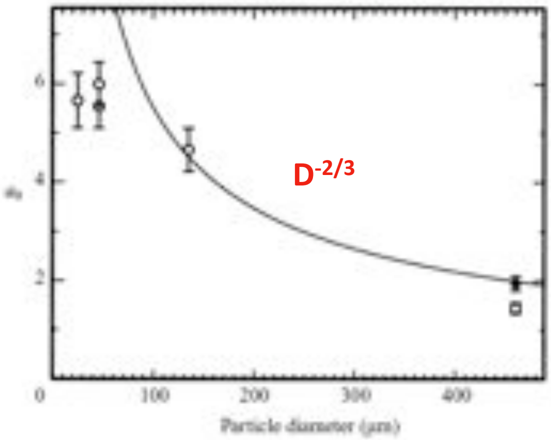
Size effect / acc. variance

$$\vec{a} \propto \vec{\nabla}_D \, p$$

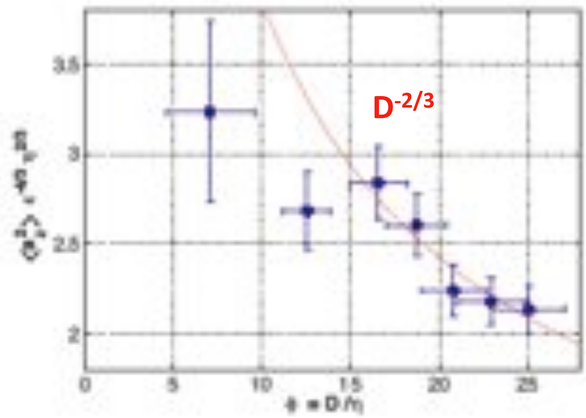
$$\downarrow$$

$$\langle a^2 \rangle = a_0' \epsilon^{4/3} D^{-2/3}$$

Voth et al. (2002)

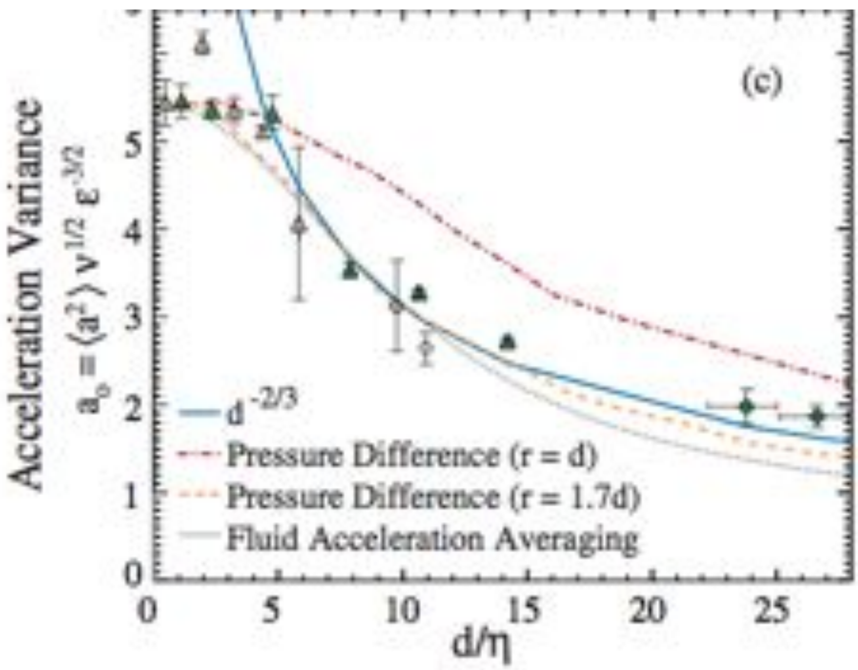


Qureshi et al. (2007)



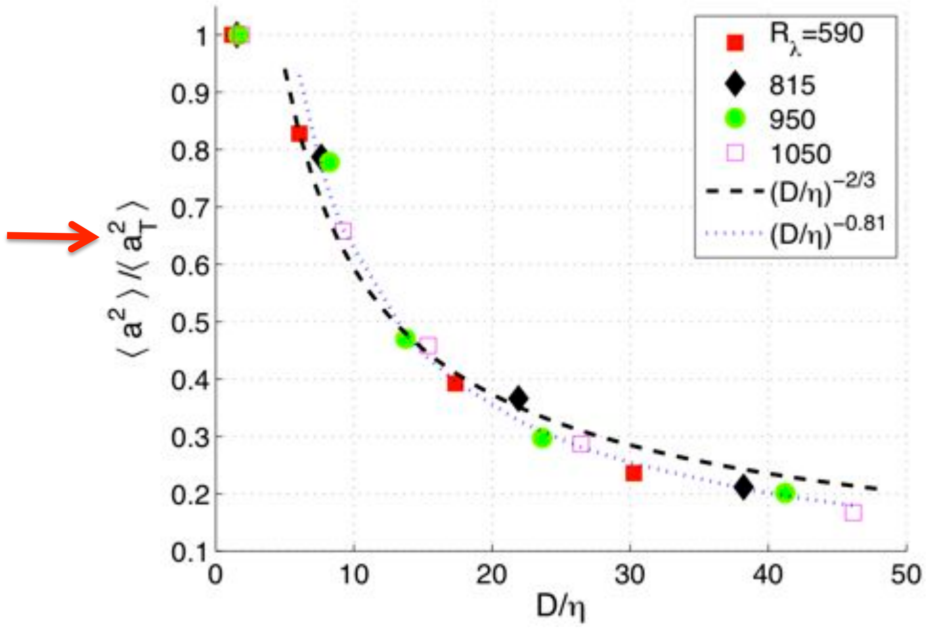
Size effect / acc. variance

Brown, Warhaft & Voth, *PRL* **103**, 194501 (2009)



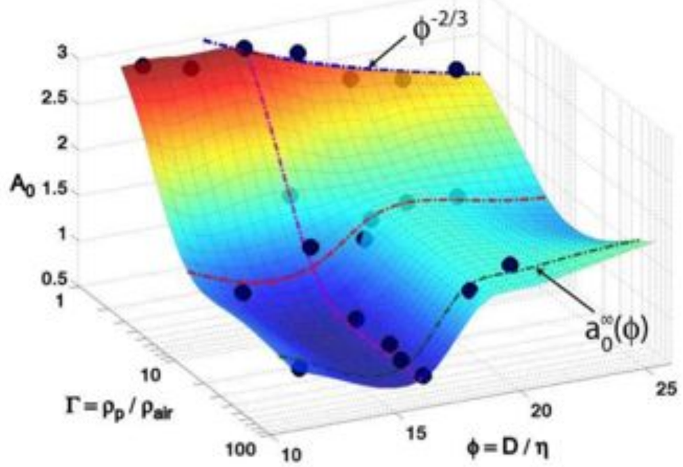
Acceleration variance

Romain Volk et al., preprint (2010)



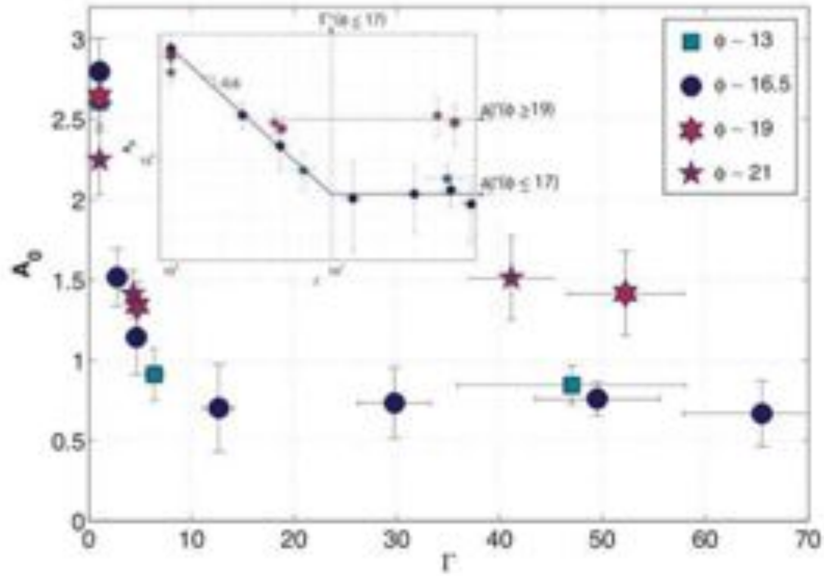
Acceleration variance

Qureshi et al., *EPJB* **66** (2008)

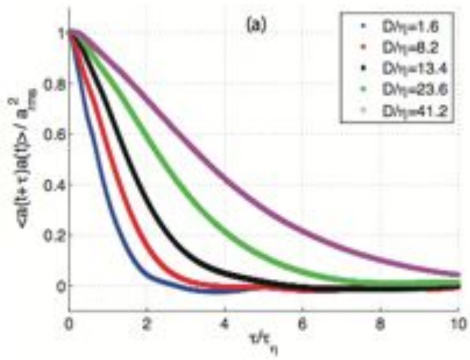


Acceleration variance

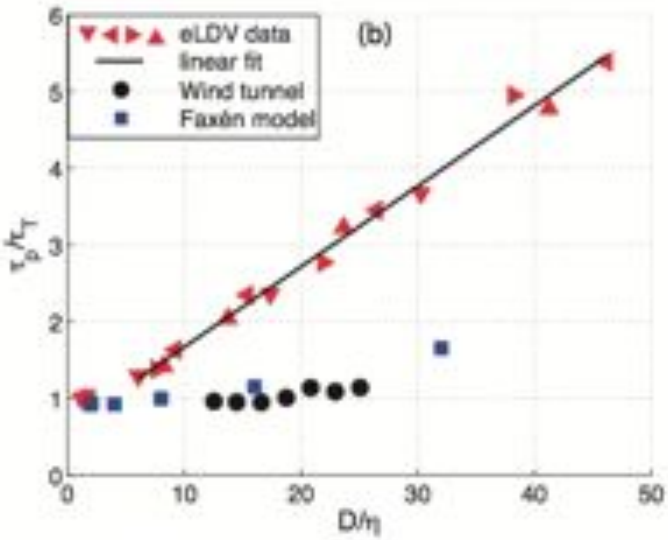
Qureshi et al., *EPJB* **66** (2008)



Response time, from <a(t)a(t+τ)>

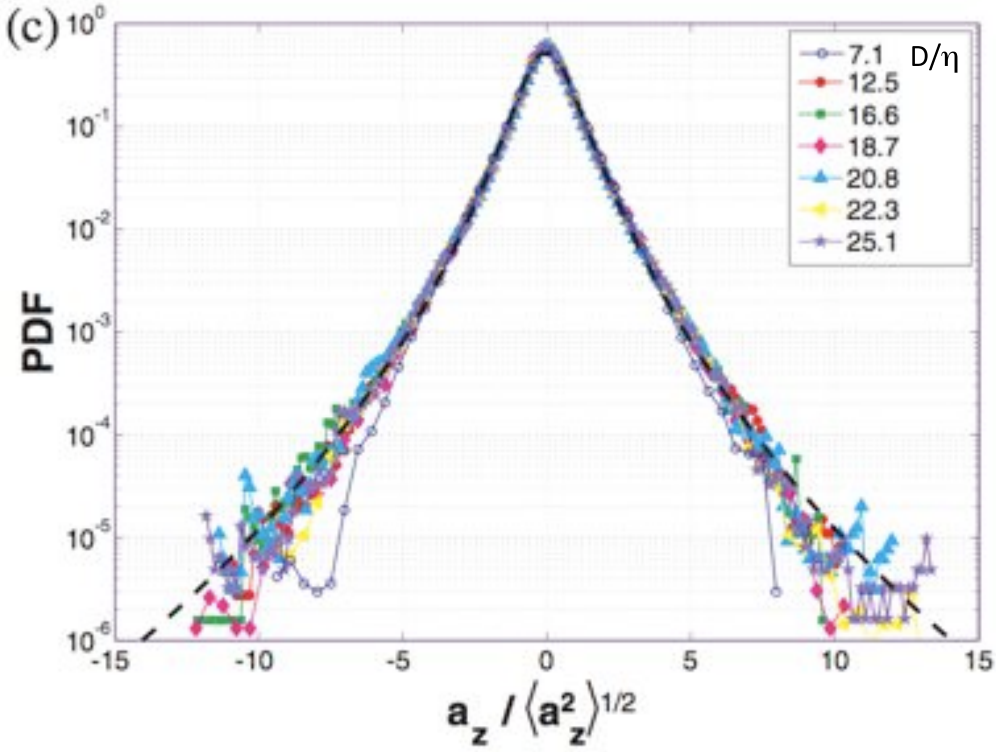


$\rho_p = \rho_f$



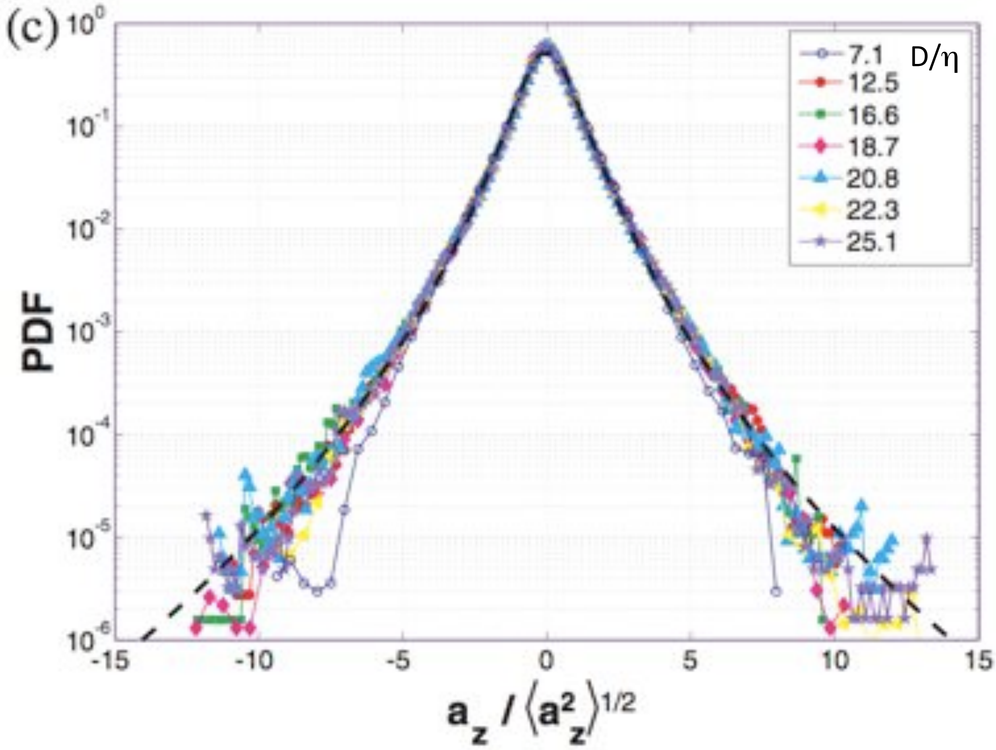
Grid flow

Qureshi, Bourgoin, Baudet, Cartellier, Gagne, *PRL* **99**, 184502 (2007)



Grid flow

Qureshi, Bourgoin, Baudet, Cartellier, Gagne, *PRL* **99**, 184502 (2007)



VK FLOW

Brown, Warhaft & Voth, *PRL* **103**, 194501 (2009)

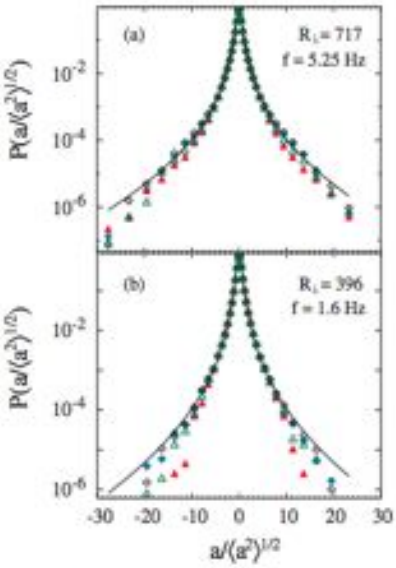
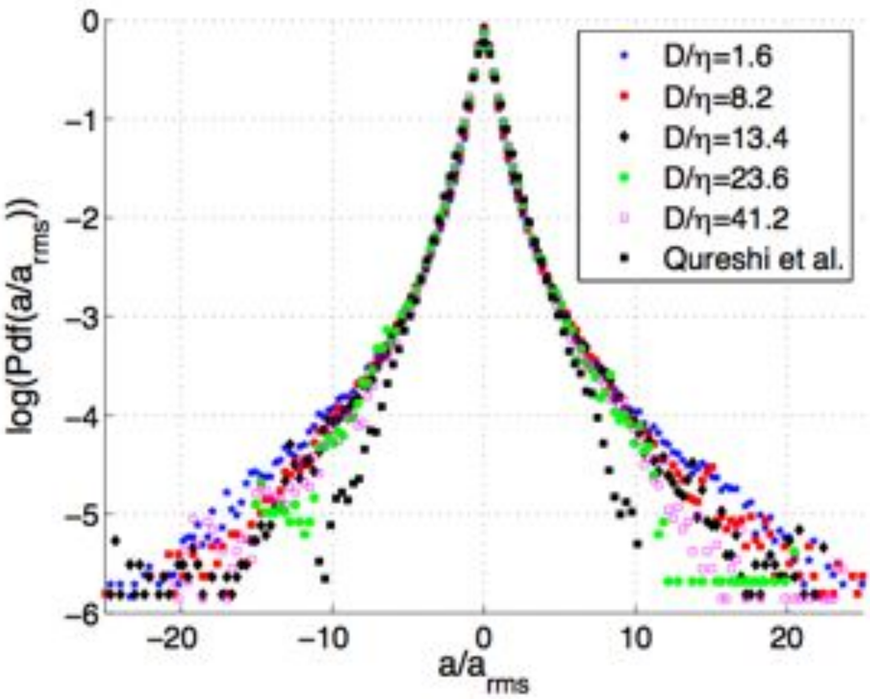
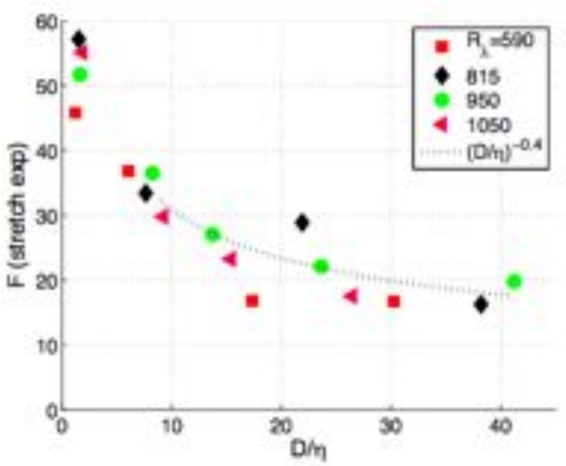


FIG. 3 (color online). Probability density functions of acceleration at Taylor Reynolds numbers (a) 717 and (b) 396. Particle sizes in d/η represented by each symbol are as follows: black unfilled diamonds, (a) 1.95 and (b) 4.75; blue filled diamonds, (a) 3.23 and (b) 7.87; green unfilled triangles, (a) 4.37 and (b) 10.64; and red filled triangles, (a) 5.82 and (b) 14.18.

VK flow



Evolution of acceleration flatness



$$\langle a^n \rangle \sim \langle \delta_D P^n \rangle / D^n$$

.

$$\langle \delta_D P \rangle \propto \langle \delta_D v^2 \rangle$$

$$\langle \delta_D P^n \rangle \propto D^{\zeta_{2n}}$$

$$\langle a^n \rangle \propto D^{\zeta_{2n}-n}$$

.

variance

$$\langle a^2 \rangle \propto D^{\zeta_4 - 2} \sim D^{-0.8}$$

flatness

$$F(D) = \langle a^4 \rangle / \langle a^2 \rangle^2 \propto D^{\zeta_8 - 2\zeta_4} \sim D^{-0.4}$$

dynamical questions

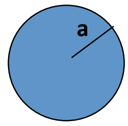
$$\rho_p \frac{d\mathbf{v}}{dt} = \rho_f \frac{D\mathbf{u}}{Dt} + (\rho_p - \rho_f)\mathbf{g}$$
$$- \frac{9\nu\rho_f}{2a^2}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right)$$
$$- \frac{\rho_f}{2}\left(\frac{d\mathbf{v}}{dt} - \frac{D}{Dt}\left[\mathbf{u} + \frac{a^2}{10}\nabla^2\mathbf{u}\right]\right)$$
$$- \frac{9\rho_f}{2a}\sqrt{\frac{\nu}{\pi}}\int_0^t \frac{1}{\sqrt{t-\zeta}}\frac{d}{d\zeta}\left(\mathbf{v} - \mathbf{u} - \frac{a^2}{6}\nabla^2\mathbf{u}\right) d\zeta$$

- acceleration (force) variance
- response time
- PDF of fluctuations

- modeling

Faxen corrected model

Calzavarini et al. JFM (2009)



Improved particle equation: non uniformity of flow at particle scale,
Particle is moving in a spatially averaged field

$$\frac{dv}{dt} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\frac{Du}{Dt} + \frac{3\nu}{a^2}(u - v)\right)$$

volume average **surface average**

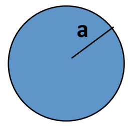
$$\frac{dv}{dt} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\left\langle\frac{Du}{Dt}\right\rangle_V + \frac{3\nu}{a^2}(\langle u\rangle_S - v)\right)$$

Maxey, M. & Riley, J. 1983
Gatignol, R. 1983

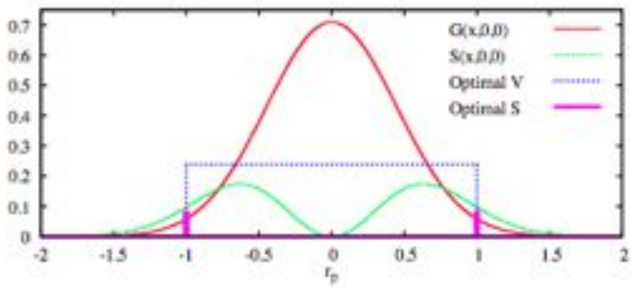
Computationally efficient
Spectral DNS + filtering in k space
(gaussian kernel)

Faxen corrected model

Calzavarini et al. JFM (2009)

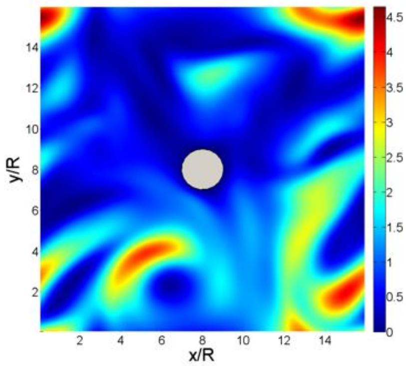


$$\frac{dv}{dt} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\left\langle\frac{Du}{Dt}\right\rangle_V + \frac{3\nu}{a^2}(\langle u\rangle_S - v)\right)$$



modeling …

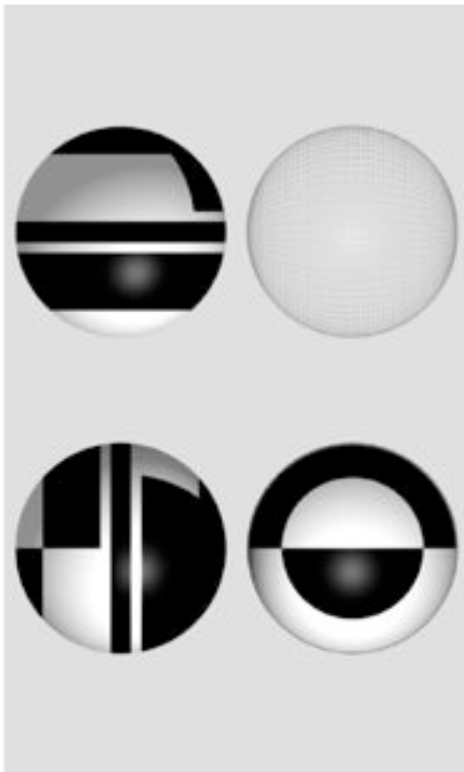
- Faxen corrections
  Calzavarini et al., *JFM* **630** (2009)
- Physalis code
  Naso & Prosperetti, *NJP* **12** (2010)
- Penalty method
  Homann & Bec, *JFM* **651** (2010)
- …



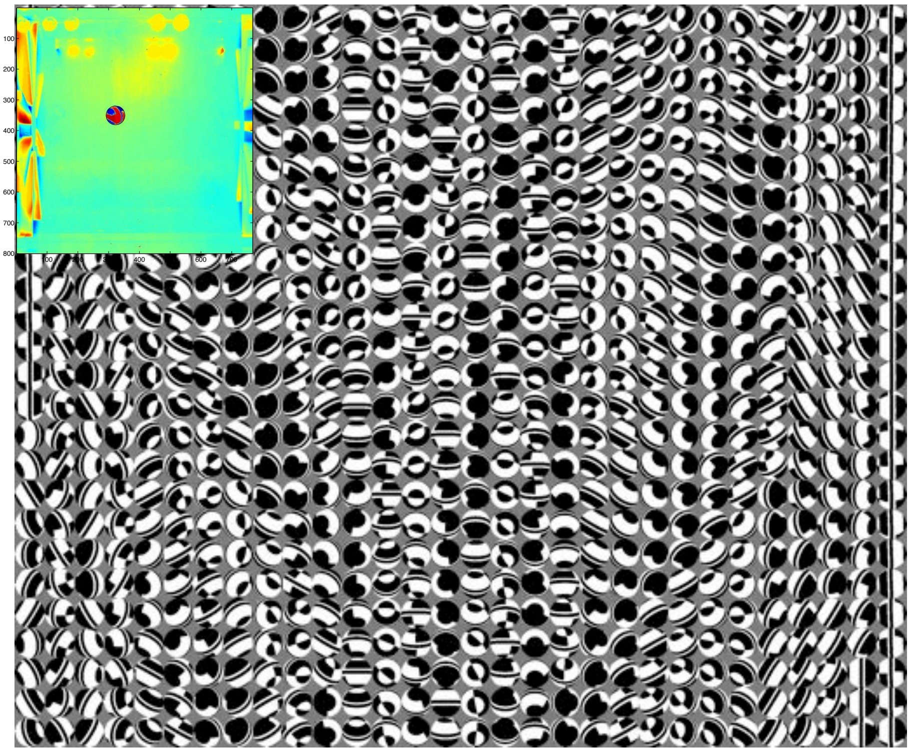
Orientations

Yoann GASTEUIL, Robert Zimmermann
PhD thesis

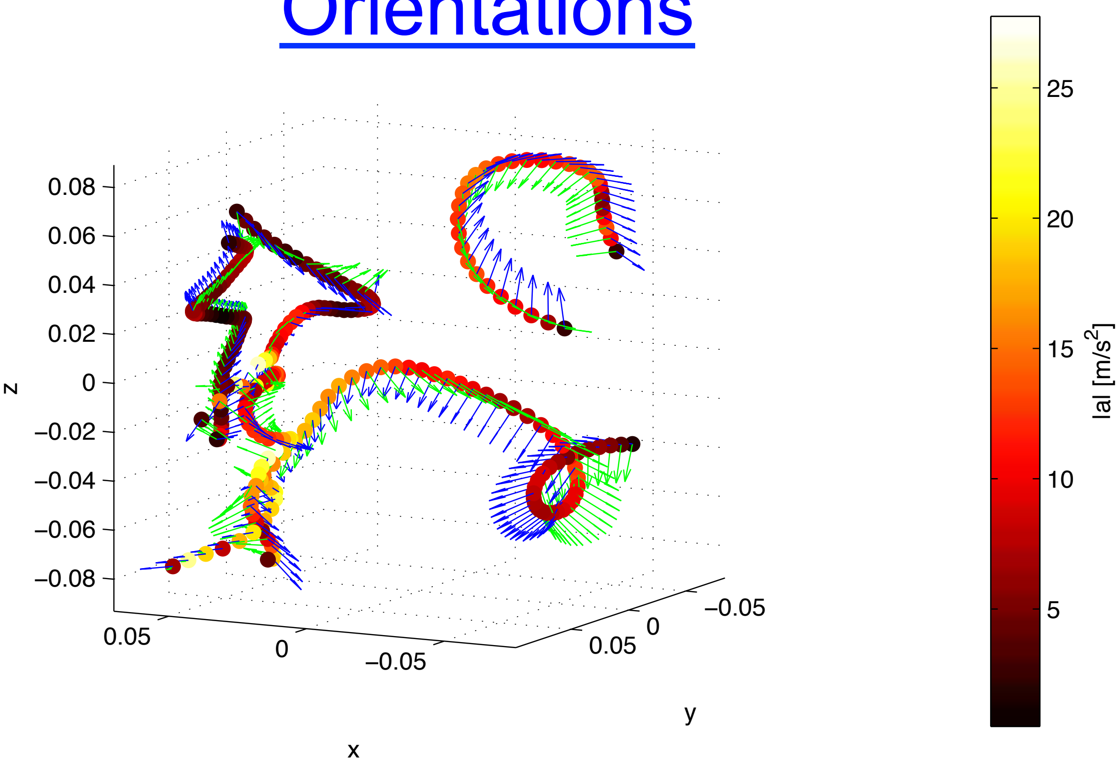
- Shadow problems
- Gimbal lock
- OpenGL on Nvida



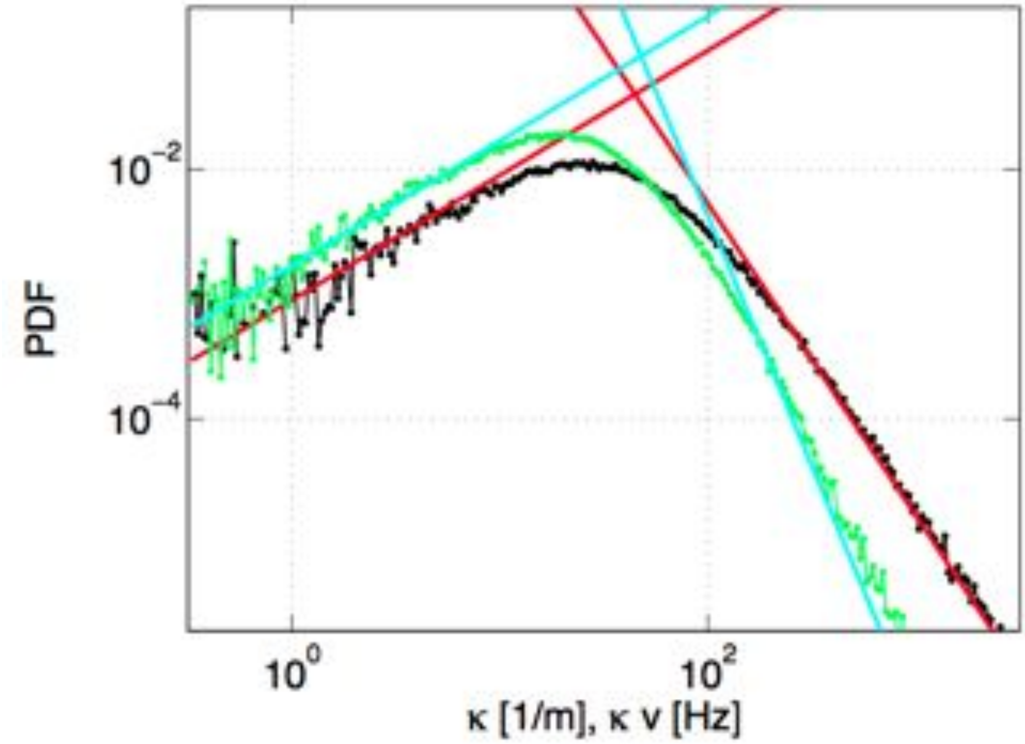
Orientations



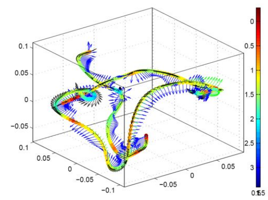
Orientations

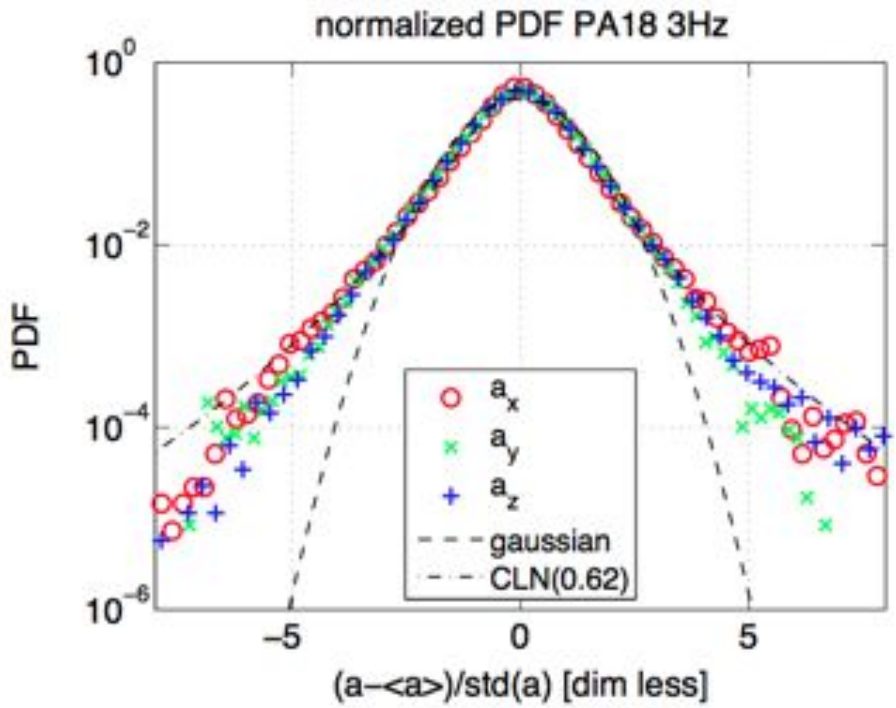


Trajectory

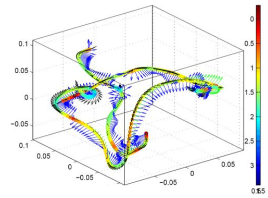


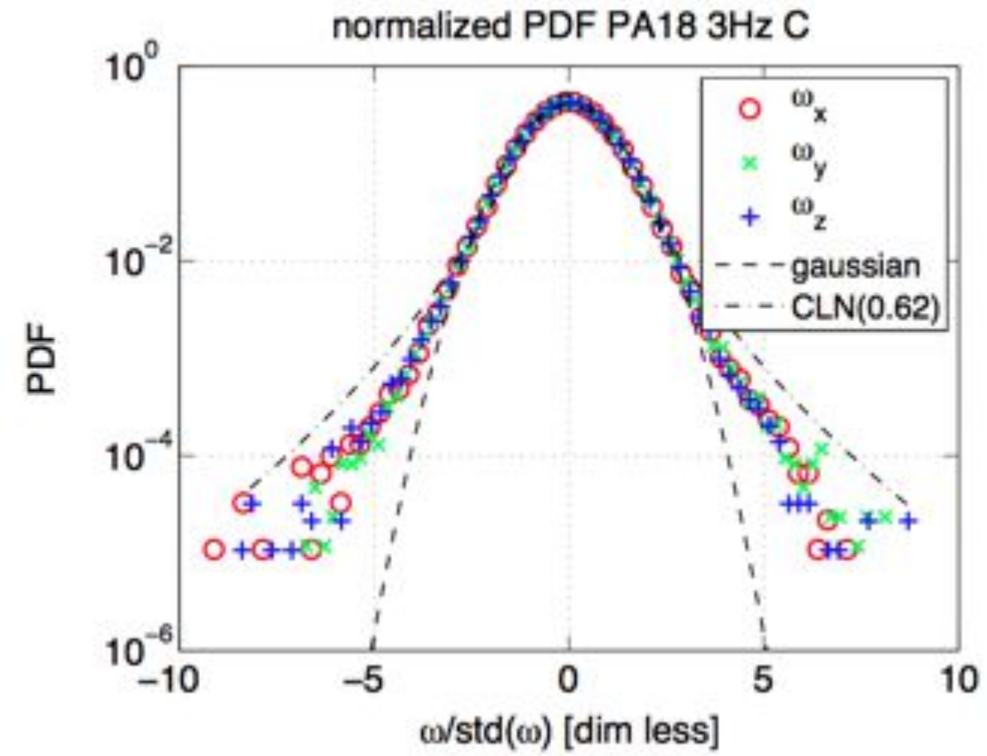
linear acceleration



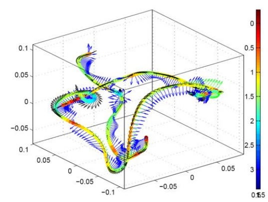


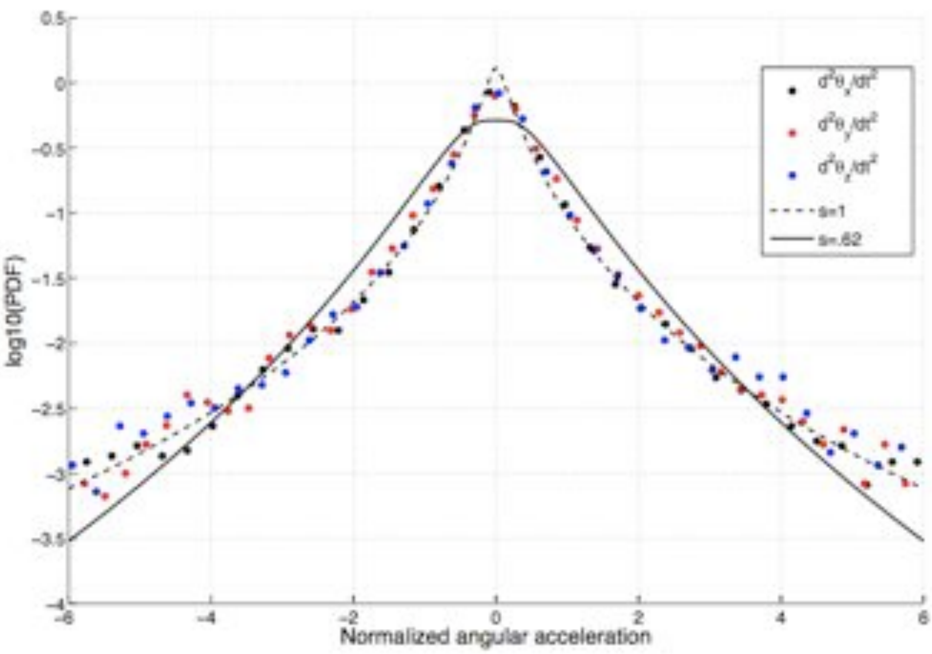
Angular velocity



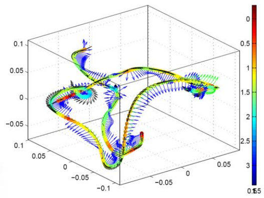


Angular acceleration



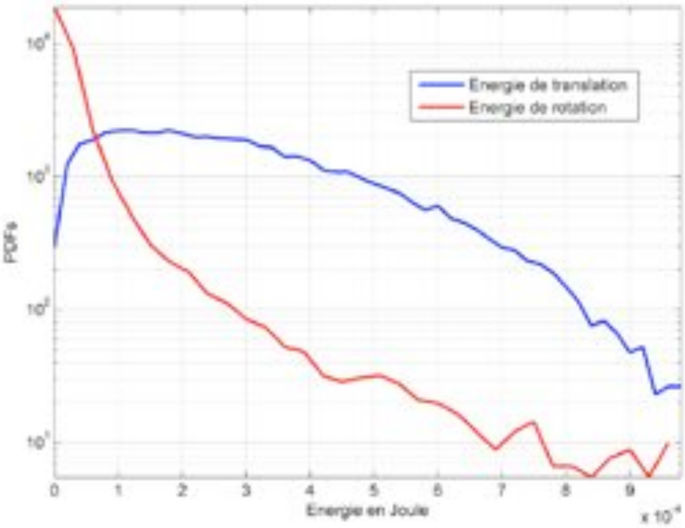


Energy budget



$$E_{k\ \mathrm{trans}} = \frac{mv^2}{2},$$

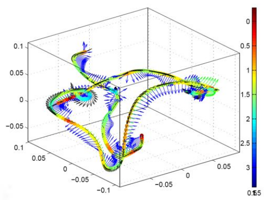
$$E_{k\ \mathrm{rot}} = \frac{Jw^2}{2},$$

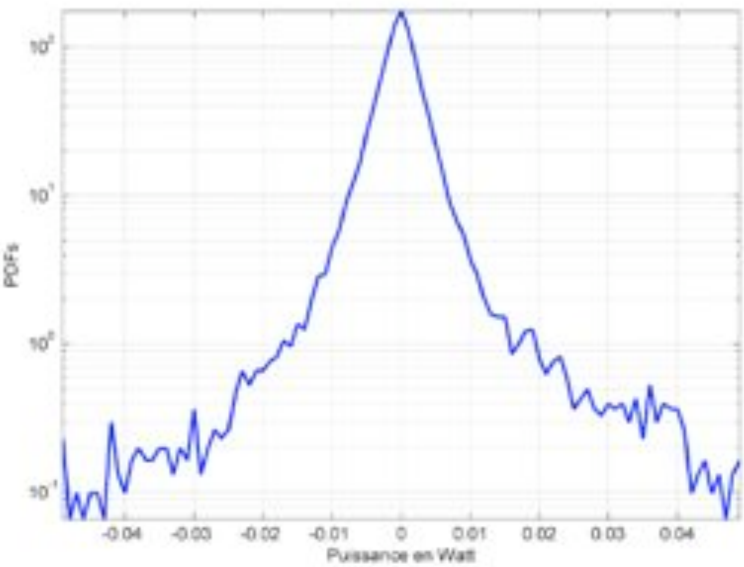


$$E_t = (1/2)mv^2,\ E_r = 1/2J(d\theta/dt)^2,\ \text{with } J = (2/5)m(D/2)^2$$

Energy	*mean*	*rms*
Ek trans	300 µJ	200 µJ
Ek rot	60 µJ	340 µJ

Energy budget



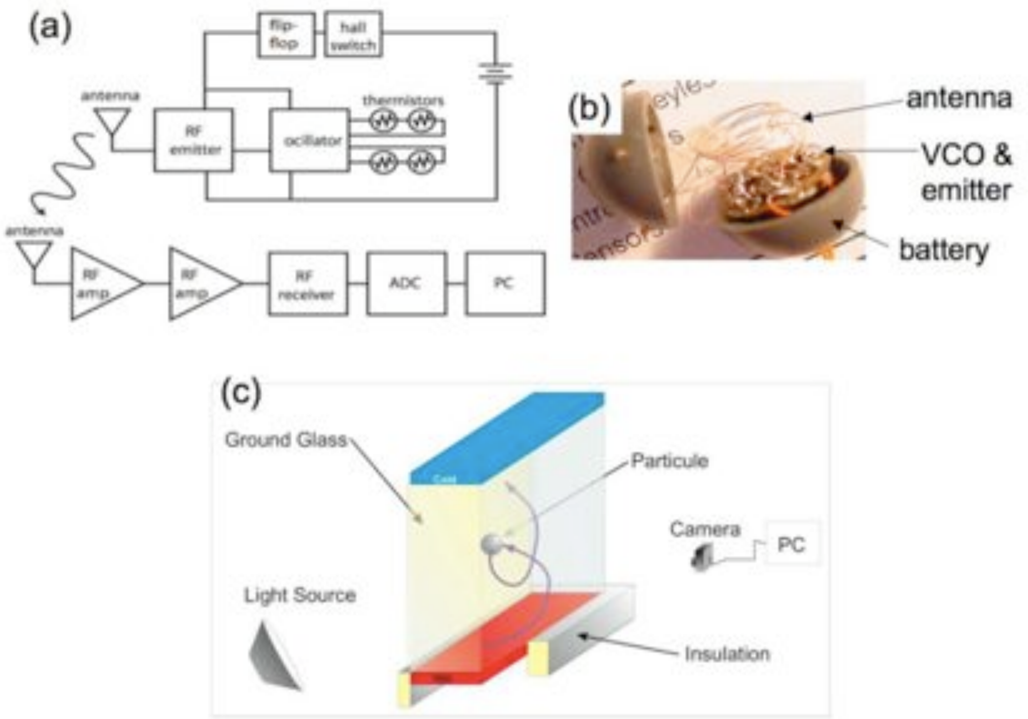


Power: $P = d(E_t + E_r)/dt,\ \overline{P} = -12\ \mu\mathrm{W},\ p_{\mathrm{rms}} = 2.1\ \mathrm{mW}$

Fluctuation / Dissipation relationships ?

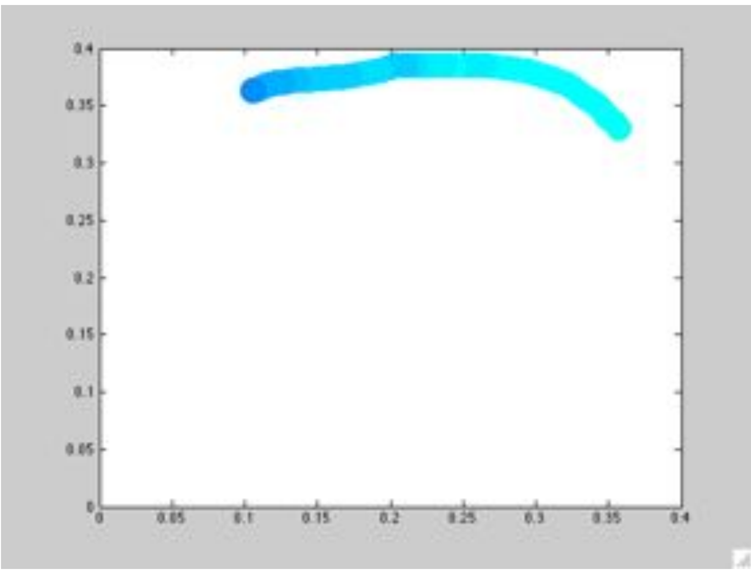
Rayleigh Bénard convection

Y. Gasteuil, M. Gibert, W. Shew, P. Metz, JFP, *Rev. Sci. Instr.* (2007)



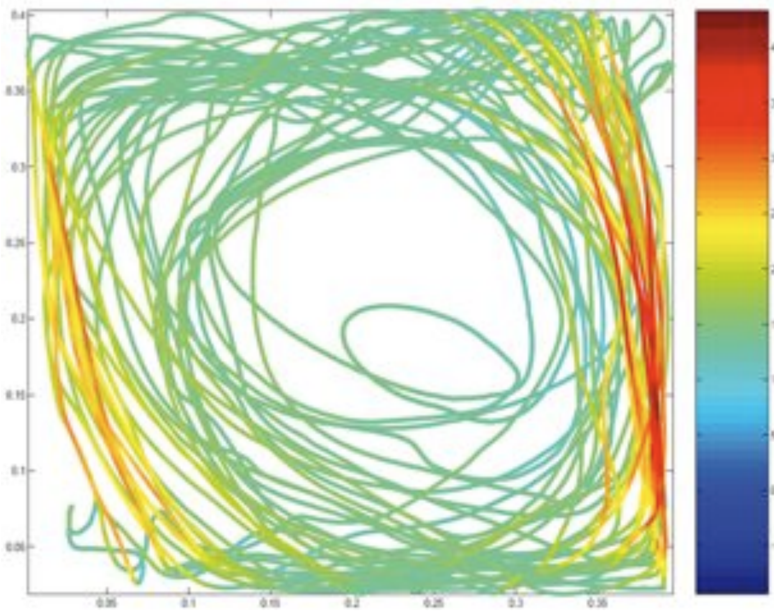
Lag. temperature in Rayleigh-Bernard

Y. Gasteuil, M. Gibert, W. Shew, P. Metz, JFP, *Rev. Sci. Instr.* (2007)



“plumes” in Rayleigh-Bernard

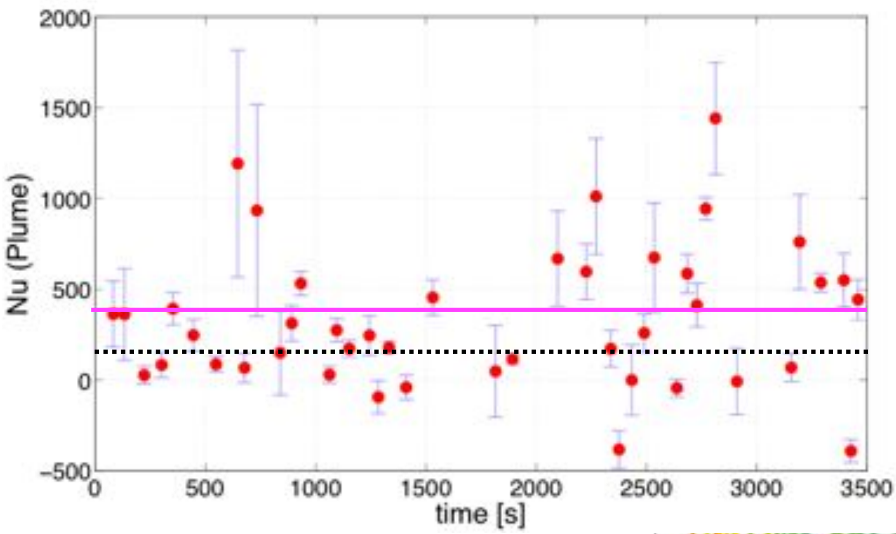
Y. Gasteuil, M. Gibert, W. Shew, P. Metz, JFP, *Rev. Sci. Instr.* (2007)

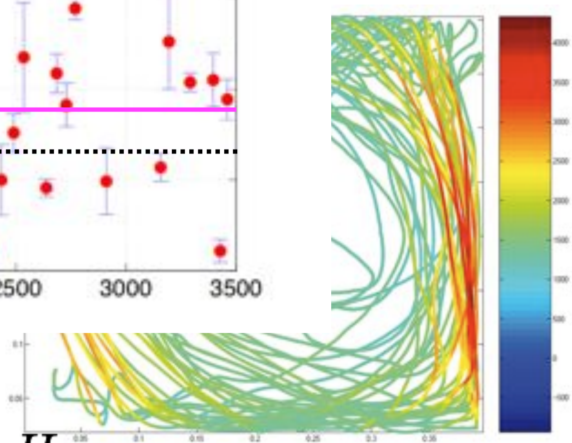


$$Nu^L(T) = 1 + \frac{H}{\kappa \Delta T} v_z(t)\theta(t)$$

“plumes” in Rayleigh-Bernard

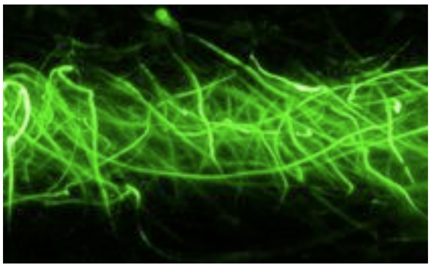
Y. Gasteuil, M. Gibert, W. Shew, P. Metz, JFP, *Rev. Sci. Instr.* (2007)

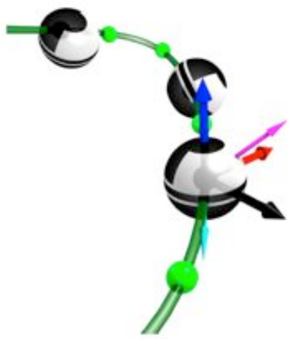




$$Nu^L(T) = 1 + \frac{H}{\kappa \Delta T} v_z(t)\theta(t)$$

"turb. mechanics 101"





Yoann Gasteuil, Romain Volk, Robert Zimmermann, Enrico Calzavarini,
Emmanuel Lévêque, Alain Pumir, Aurore Naso,
Nicolas Mordant, Pascal Metz, Gautier Verhille,

laboratoire de physique