QUASI-GEOSTROPHIC TURBULENCE

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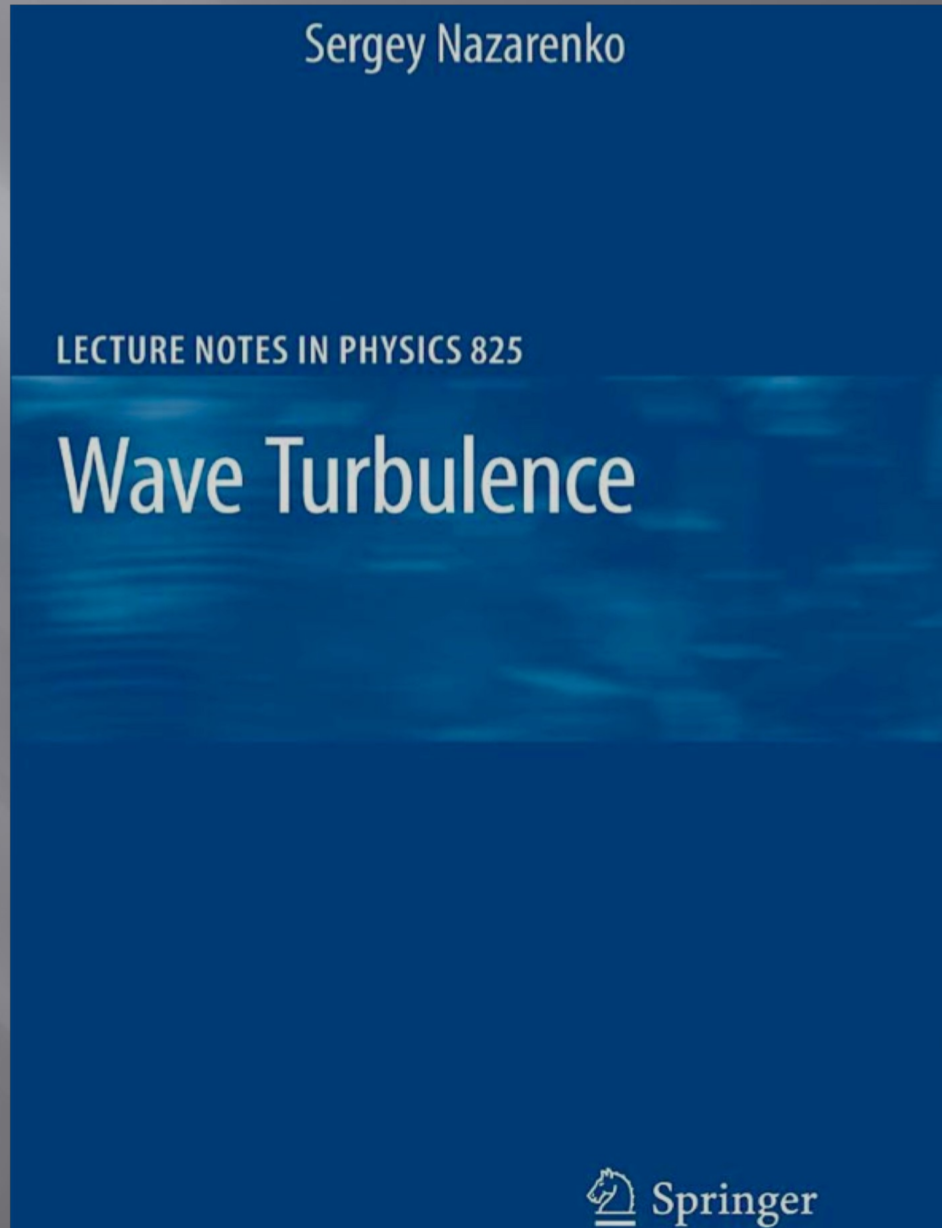
Overview of work in collaboration with Balk, Bustamante, Connaughton, Dyachenko, Harper, Manin, Medvedev, Nadiga, Quinn, Zakharov

AGAT2016. 25 July to 5 August 2016

Outline

- Importance of resonant wave interactions in QG turbulence
- Generation of zonal jets by local anisotropic cascades and nonlocal mechanisms.
- Quadratic invariants.
- Self-regulating turbulence – zonal jet system
- Continuous spectrum v discrete-wave clusters

A chapter on Rossby wave turbulence in:



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Rossby and drift wave turbulence and zonal flows: The Charney–Hasegawa–Mima model and its extensions

Rossby and drift wave turbulence and zonal flows: The Charney–Hasegawa–Mima model and its extensions

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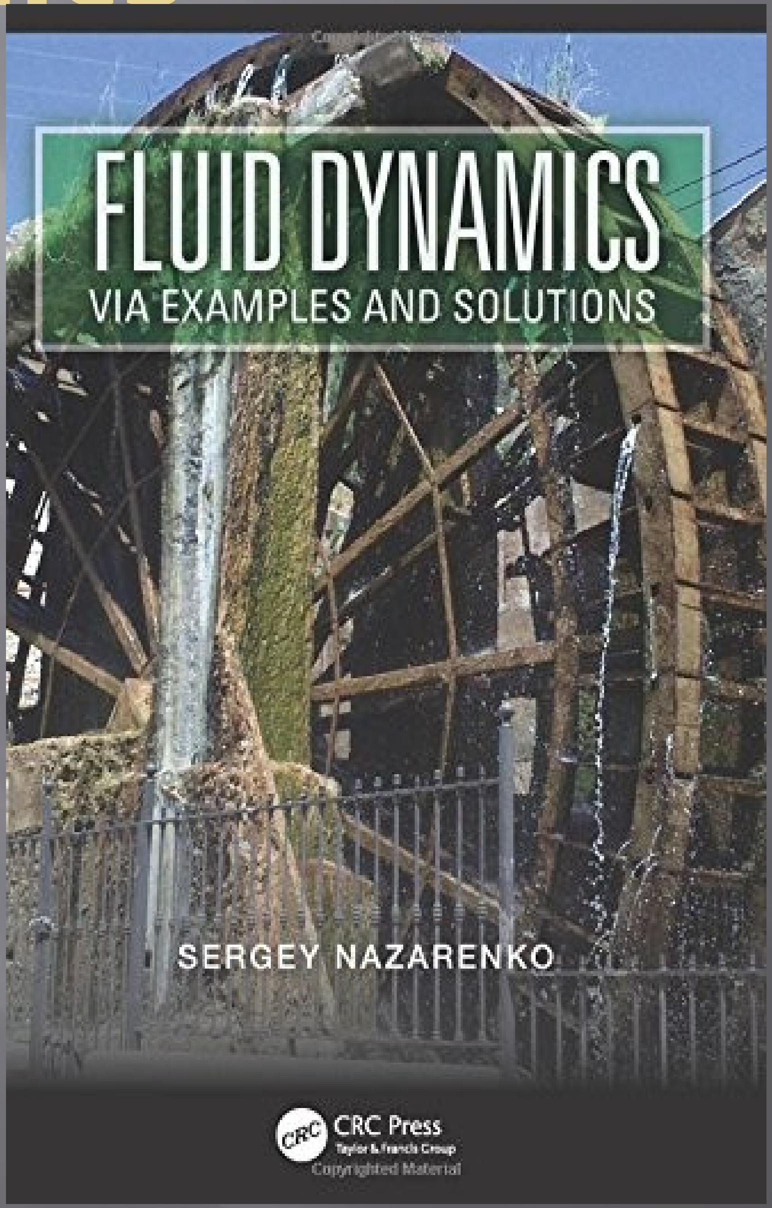
Zonal Jets

The ISSI Team

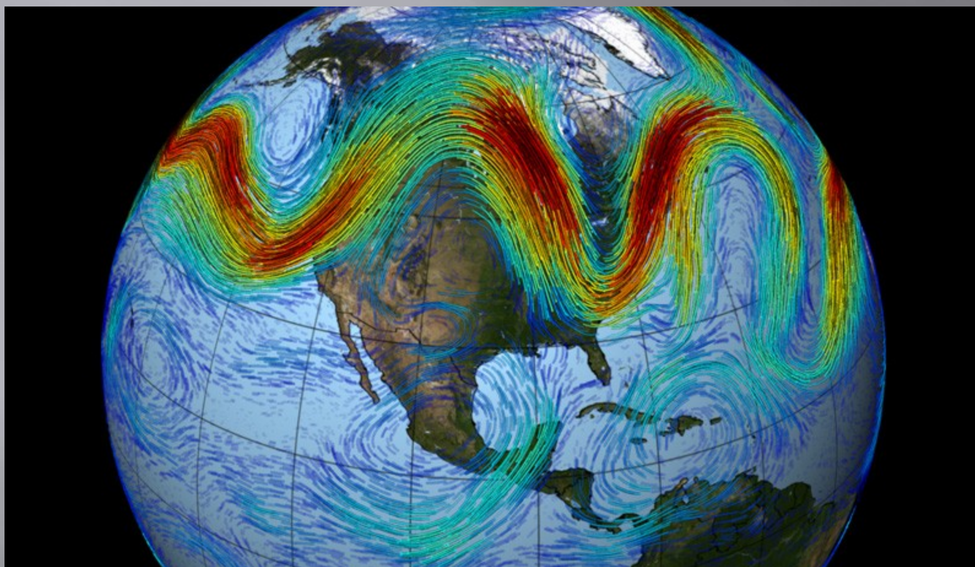
Editors: Boris Galperin and Peter Read

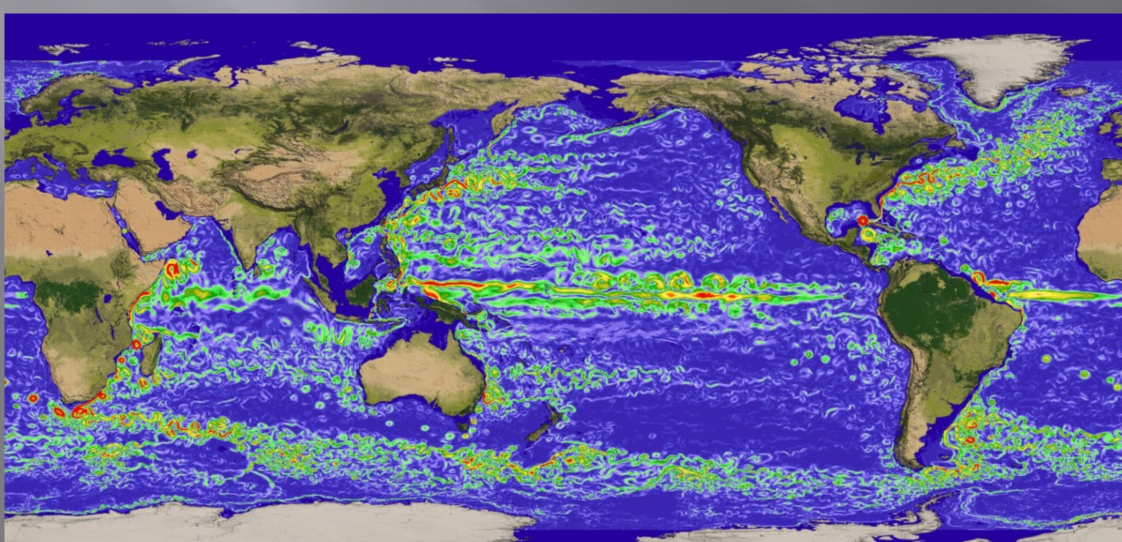
UG text in Fluid Dynamics



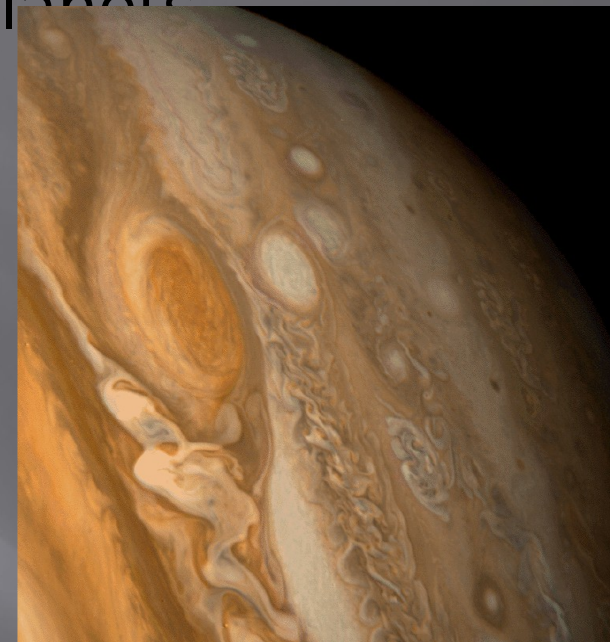
Rossby waves and jets

Earth's atmosphere and ocean





Atmospheres of giant planets

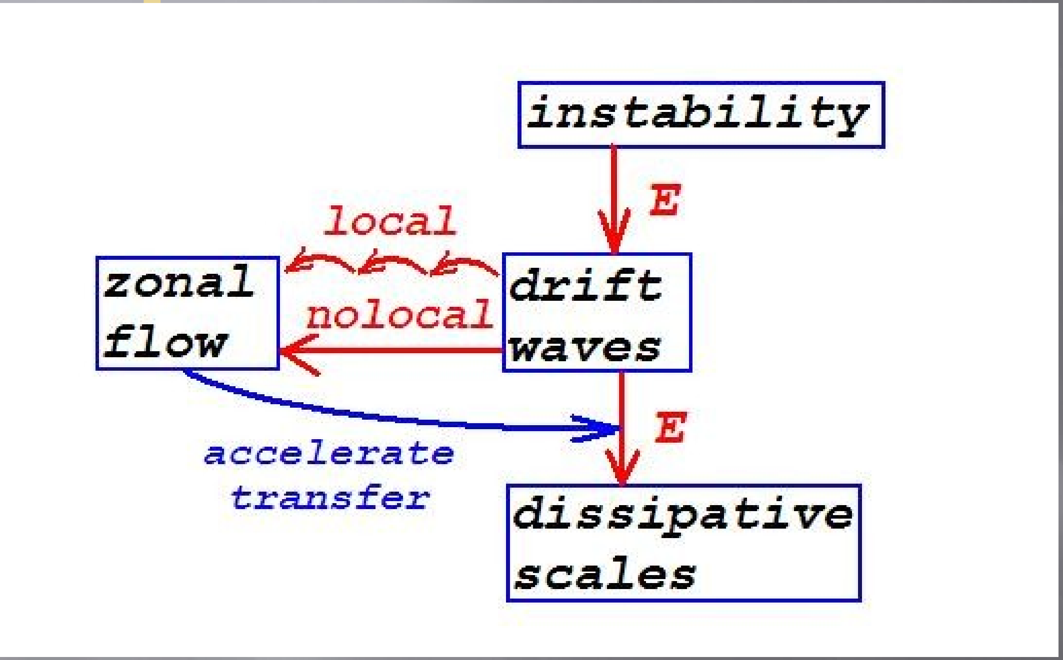


CHARNEY-HASEGAWA-MIMA EQUATION

$$\frac{\partial}{\partial t}\left(\rho^2\nabla^2\psi-\psi\right)-\beta\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}=0$$

- Ψ – streamfunction (electrostatic potential).
- ρ – Deformation radius (ion Larmor radius).
- β – PV gradient (diamagnetic drift).
- x – east-west (poloidal arc-length)
- y – south-north (radial length).

Turbulence/Zonal-Flow feedback loop



Balk, SN and Zakharov 1990

- Small-scale turbulence generates zonal flows.
- Negative feedback loop: turbulence is suppressed by ZFs
- Suppressed turbulence → reduced anomalous transport

Barotropic governor in GFG

- James and Gray' 1986

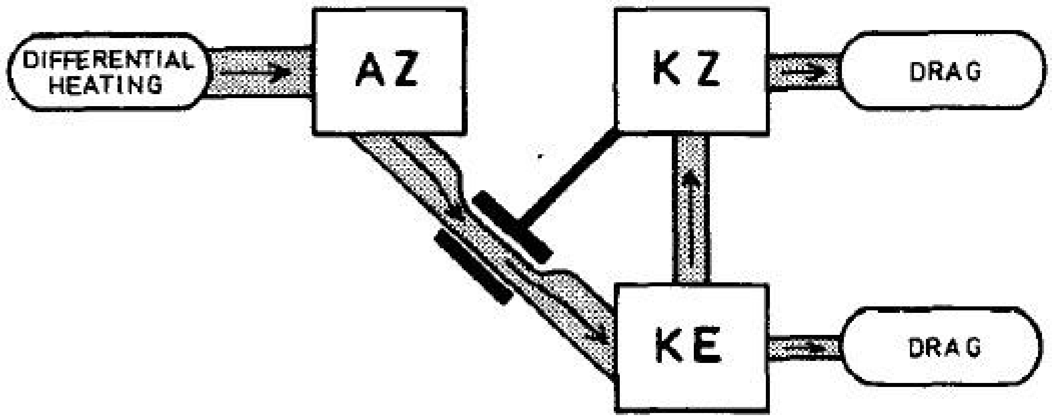
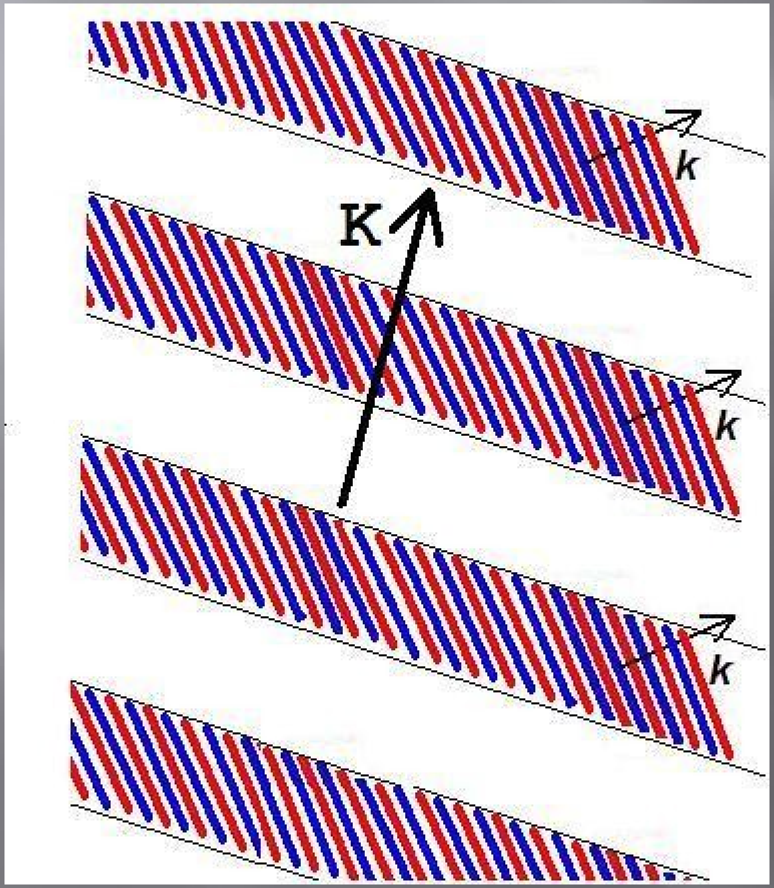


FIG. 1. Schematic illustration of the "barotropic governor," summarizing the effect of horizontal shears on baroclinic instability as postulated in JG. Energy conversion from available potential energy (AZ) to eddy kinetic energy (KE) results in momentum fluxes which increase the barotropic contribution to the zonal kinetic energy (KZ). As barotropic shears build up in the zonal flow, the baroclinic conversions are inhibited.

Nonlocal mechanism of ZF generation: Modulational Instability

Loretz 1972, Gill 1973, Manin, Nazarenko, 1994
Numerics: Connaughton, Nadiga, SN, Quinn, 2009.



- ***Cf. Benjamin-Fair Instability of water waves***

Modulational Instability

$$\psi_0(\mathbf{x},t) = \Psi_0 e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \bar{\Psi}_0 e^{-i\mathbf{k}\cdot\mathbf{x}+i\omega t}$$

$$\omega(\mathbf{k}) = -\frac{\beta k_x}{k^2+F}$$ – frequency of linear waves.

- These waves are solutions of CHM equation for any amplitude. Are they stable? (Lorentz 1972, Gill 1973).

$$\psi(\mathbf{x},0) = \psi_0(\mathbf{x}) + \varepsilon\psi_1(\mathbf{x}),$$

$$\psi_1(\mathbf{x}) = \psi_Z(\mathbf{x}) + \psi^+(\mathbf{x}) + \psi^-(\mathbf{x})$$ – perturbation.

$$\psi_Z(\mathbf{x}) = a e^{i\mathbf{q}\cdot\mathbf{x}} + \bar{a} e^{-i\mathbf{q}\cdot\mathbf{x}}$$ – zonal part $\mathbf{q} = (0, q)$,

$$\psi^+(\mathbf{x}) = b^+ e^{i\mathbf{p}_+\cdot\mathbf{x}} + \bar{b}^+ e^{-i\mathbf{p}_+\cdot\mathbf{x}}$$ – $^+$satellite $\mathbf{p}_+ = \mathbf{k} + \mathbf{q}$,

$$\psi^-(\mathbf{x}) = b^- e^{i\mathbf{p}_-\cdot\mathbf{x}} + \bar{b}^- e^{-i\mathbf{p}_-\cdot\mathbf{x}}$$ – $^-$satellite $\mathbf{p}_- = \mathbf{k} - \mathbf{q}$.

Instability dispersion relation

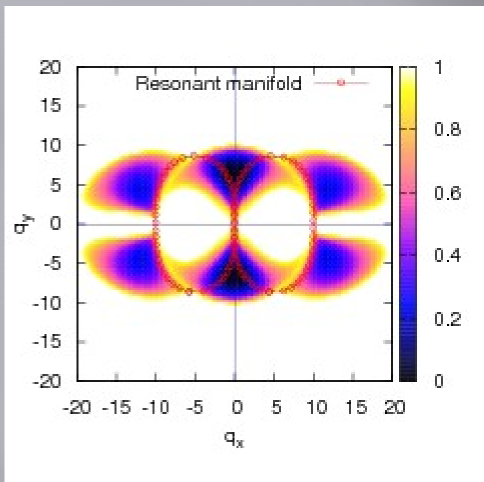
$$(q^2+F)\Omega+\beta q_x+\left|\Psi_0\right|^2\left|\mathbf{k}\times\mathbf{q}\right|^2(k^2-q^2)\left[\frac{p_+{}^2-k^2}{(p_+{}^2+F)(\Omega+\omega)+\beta p_{+x}}-\frac{p_-{}^2-k^2}{(p_-{}^2+F)(\Omega-\omega)+\beta p_{-x}}\right]=0$$

$$M=\frac{\Psi_0 k^3}{\beta}$$ – nonlinearity parameter.

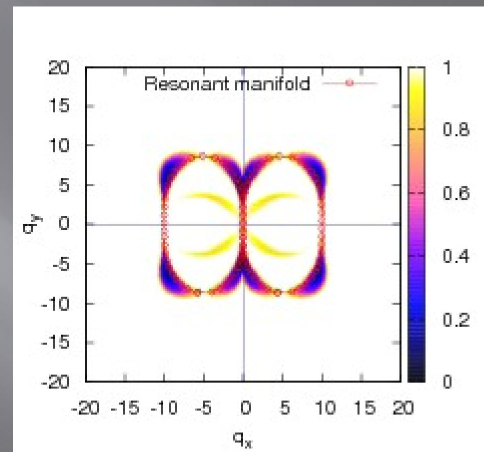
$M\to\infty$ – Euler limit (Rayleigh instabilty);

$M\to 0$ – weak monlinearity: resonant wave inetraction.

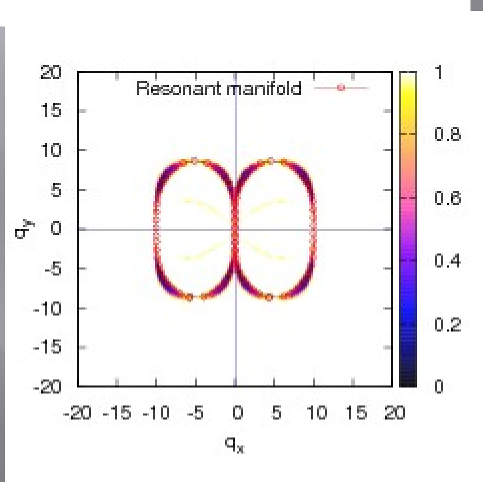
Structure of instability as a function of M



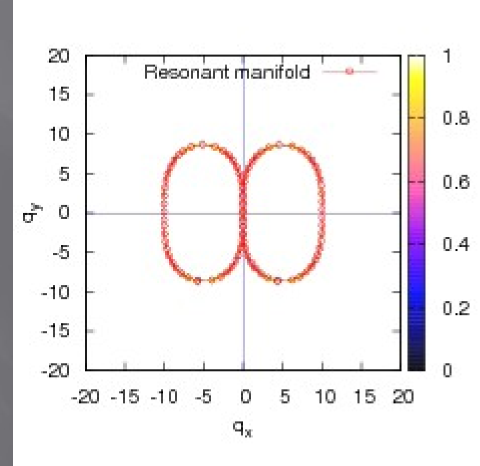
M=10



M=1



M=0.5



M=0.1

$$\mathbf{k} + \mathbf{k}_1 = \mathbf{k}_2,$$
$$\omega(\mathbf{k}) + \omega(\mathbf{k}_1) = \omega(\mathbf{k}_2)$$

For small M the unstable region collapses onto the resonant curve and the most unstable disturbance is not zonal.

Continuous spectrum theory: Kinetic equation for weakly nonlinear Rossby waves
(Longuet-Higgens &Gill, 1967)

$$\dot{n}_k = \int |V_{12k}|^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})\delta(\omega(\mathbf{k}_1) + \omega(\mathbf{k}_2) - \omega(\mathbf{k})) \times$$
$$[n(\mathbf{k}_1)n(\mathbf{k}_2) - 2n(\mathbf{k})n(\mathbf{k}_1)\,\mathrm{sign}\,(k_x k_{1x})]\; d\mathbf{k}_1 d\mathbf{k}_2,$$

$$\omega(\mathbf{k}) = -\beta k_x / k^2,$$ - frequency

$$V_{12k} = |k_x k_{1x} k_{2x}|^{1/2} \left(\frac{k_{1y}}{k_1^2} + \frac{k_{2y}}{k_2^2} - \frac{k_y}{k^2} \right)$$ - interaction coefficient

$$n(\mathbf{k}) = k^4 |\hat{\psi}_k|^2 / (\beta k_x),$$ - waveaction spectrum

For case kρ >>1. Resonant three-wave interactions.

Conservation laws in 2D

$$E(\vec{k}) = \int \langle \vec{u}(\vec{x}+\vec{r})\cdot\vec{u}(\vec{x})\rangle e^{-i\vec{k}\cdot\vec{x}} d\vec{r}$$ - energy spectrum

$$\langle u^2 \rangle = \int E(\vec{k}) d\vec{k}$$ - energy

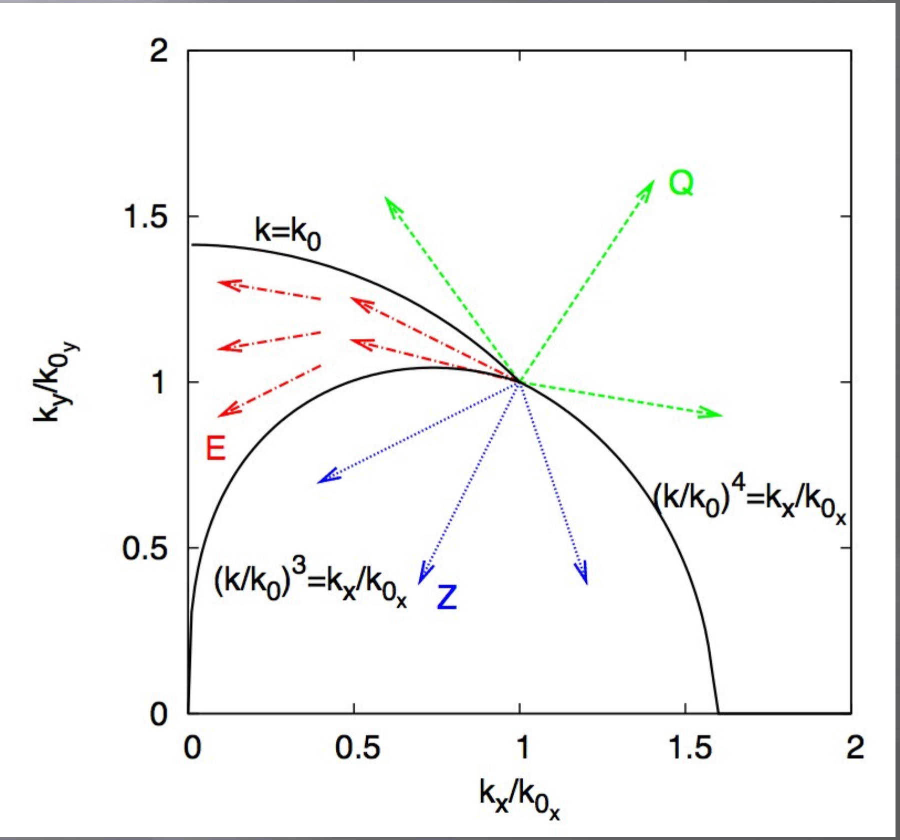
$$\langle (\nabla \times \vec{u})^2 \rangle = \int k^2 E(\vec{k}) d\vec{k}$$ -enstrophy

Extra quadratic invariant on β-plane

- *Balk, Nazarenko & Zakharov (1990)*
- *Adiabatic for the original β-plane equation: requires small nonlinearity.*
- *For case kρ >>1:*

$$\Phi = \int \frac{k_x^2}{k^6}(k_x^2 + 5k_y^2)|\hat{\psi}_k|^2 d\mathbf{k}, \quad \text{- Zonostrophy invariant.}$$

LOCAL MECHANISM OF ZONAL FLOW GENERATION
ANISOTROPIC CASCADES OF 3 INVARIANTS

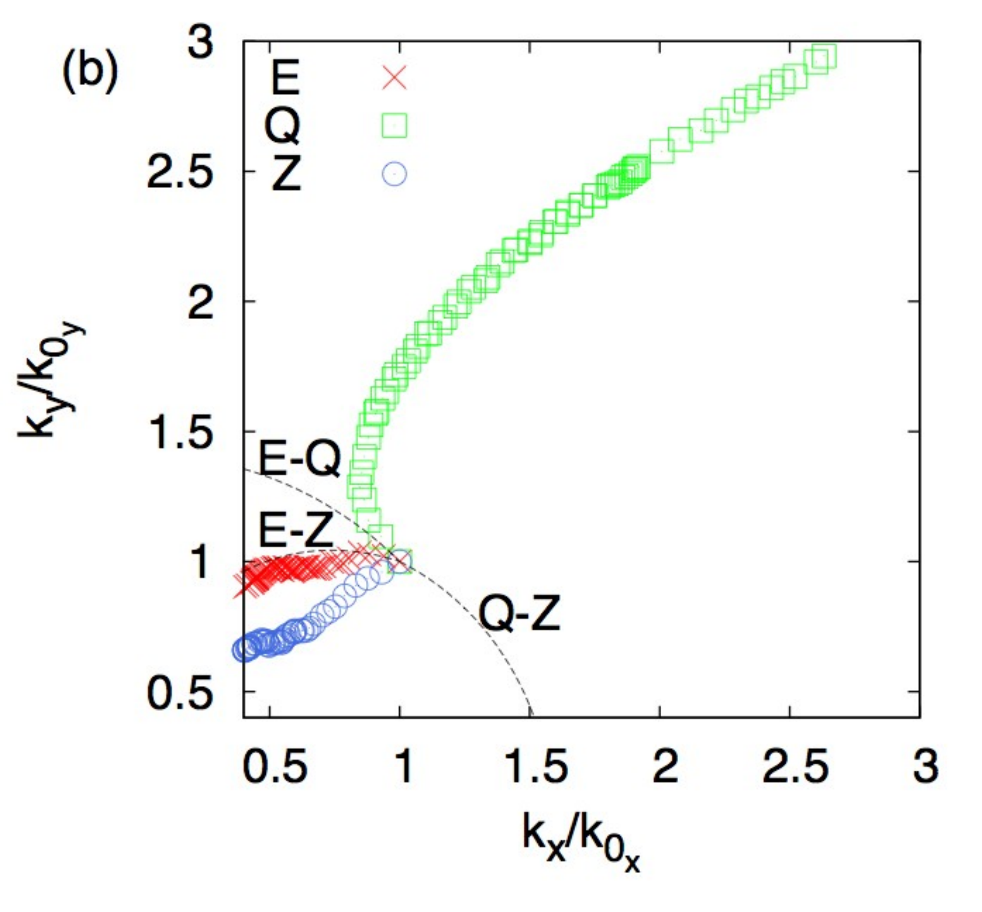


Energy flows into the zonal flow sector

Generalised Fjortoft's Theorem

- Consider a statistically steady state in a forced-dissipated system which has (in absence of forcing and dissipation) positive quadratic invariants I1, I2, …, In. Let forcing be in vicinity of k0=(k0x, ky0). The dissipation rate of Im in the regions where its relative spectral density (w.r.t. to the one at k0) is vanishingly small compared to the relative spectral density of at least one other invariant is vanishingly small w.r.t. to its production rate.
- No assumption about locality of interactions, nor about continuity or discreteness of the k-spectrum.

TRIPLE CASCADE IN QG TURBULENCE: NUMERICS OF UNFORCED CHM

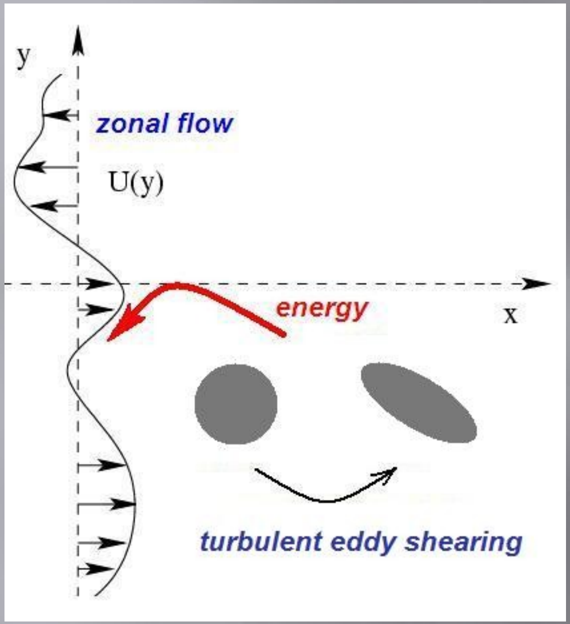


- SN and B.Quinn, 2009.
- Trajectories of the 3 centroids.
- Fjortoft works well even for strong turbulence

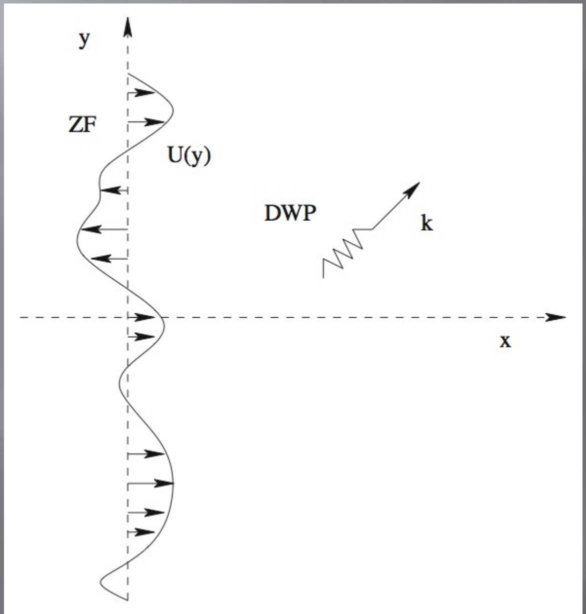
Self-regulation and Feedback loop in QG turbulence

- Instability generates small-scale turbulence.
- Inverse cascade leads to energy condensation into zonal jets.
- Jets kill small-scale turbulence and saturate.

Cartoon of ZF-turbulence nonlocal interaction

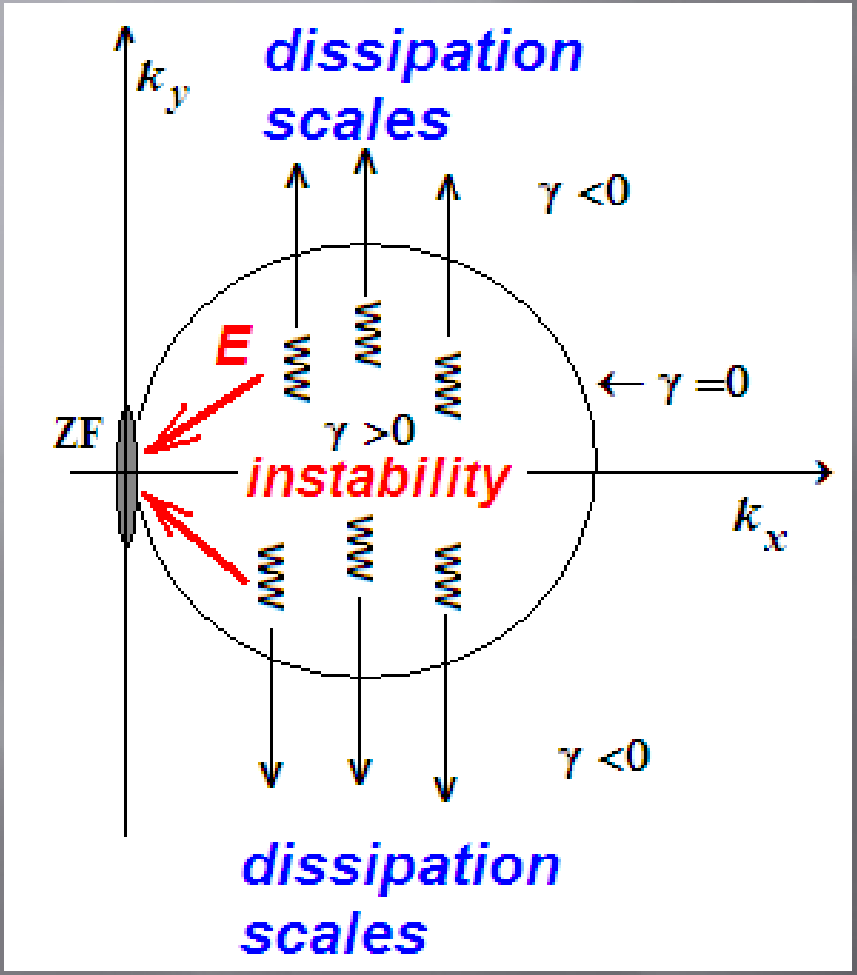


Victor P. Starr, *Physics of Negative Viscosity Phenomena* (1968).



Rossby wave turbulence.
More important for large betas

Evolution in the k-space



Energy of Rossby wave packets is partially transferred to ZF and partially dissipated at large ***k***'s.
(*Balk et al, 1990*).

Kinetic equation for weakly nonlinear Rossby/Drift waves

(Longuet-Higgens &Gill, 1967)

$$\dot{n}_k = \int |V_{12k}|^2 \,\delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})\delta(\omega(\mathbf{k}_1) + \omega(\mathbf{k}_2) - \omega(\mathbf{k})) \times$$

$$[n(\mathbf{k}_1)n(\mathbf{k}_2) - 2n(\mathbf{k})n(\mathbf{k}_1)\,\mathrm{sign}\,(k_x k_{1x})] \; d\mathbf{k}_1 d\mathbf{k}_2,$$

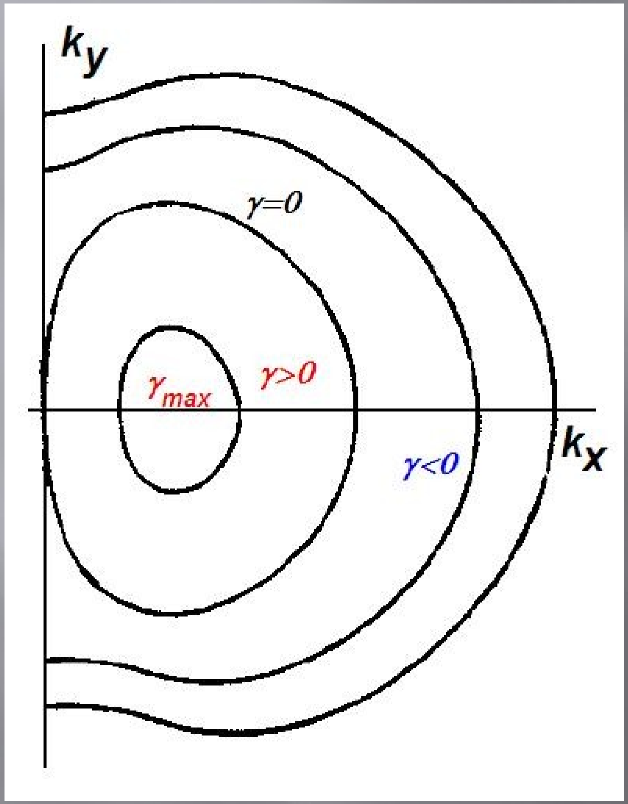
$$\omega(\mathbf{k}) = -\beta k_x / k^2,$$ - frequency

$$V_{12k} = |k_x k_{1x} k_{2x}|^{1/2} \left(\frac{k_{1y}}{k_1^2} + \frac{k_{2y}}{k_2^2} - \frac{k_y}{k^2} \right)$$ - interaction coefficient

$$n(\mathbf{k}) = k^4 |\hat{\psi}_k|^2 / (\beta k_x),$$ - waveaction spectrum

Resonant three-wave interactions.

Baroclinic instability forcing

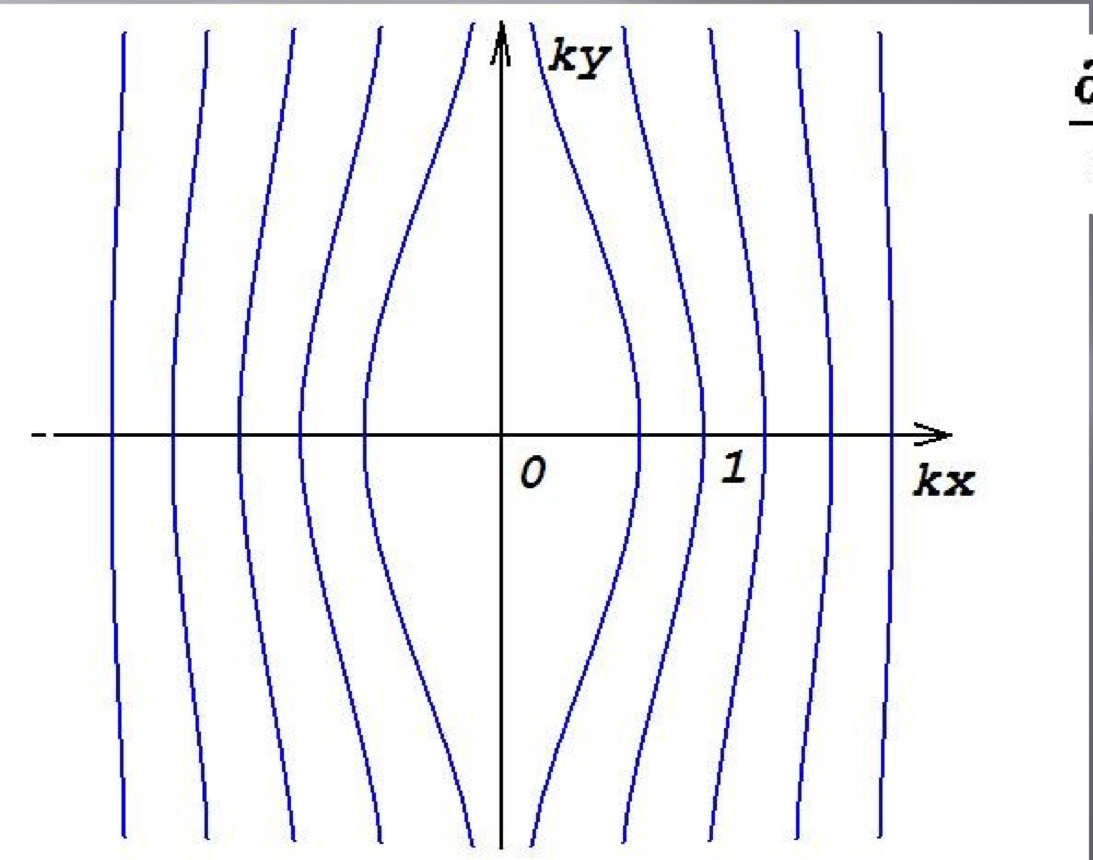


Accessing the stored free energy via instability

Maximum on the k_x-axis at $k\rho \sim 1$.

Evolution of nonlocal drift turbulence:

retain only interaction with small k's and Taylor-expand the integrand of the wave-collision integral; integrate.



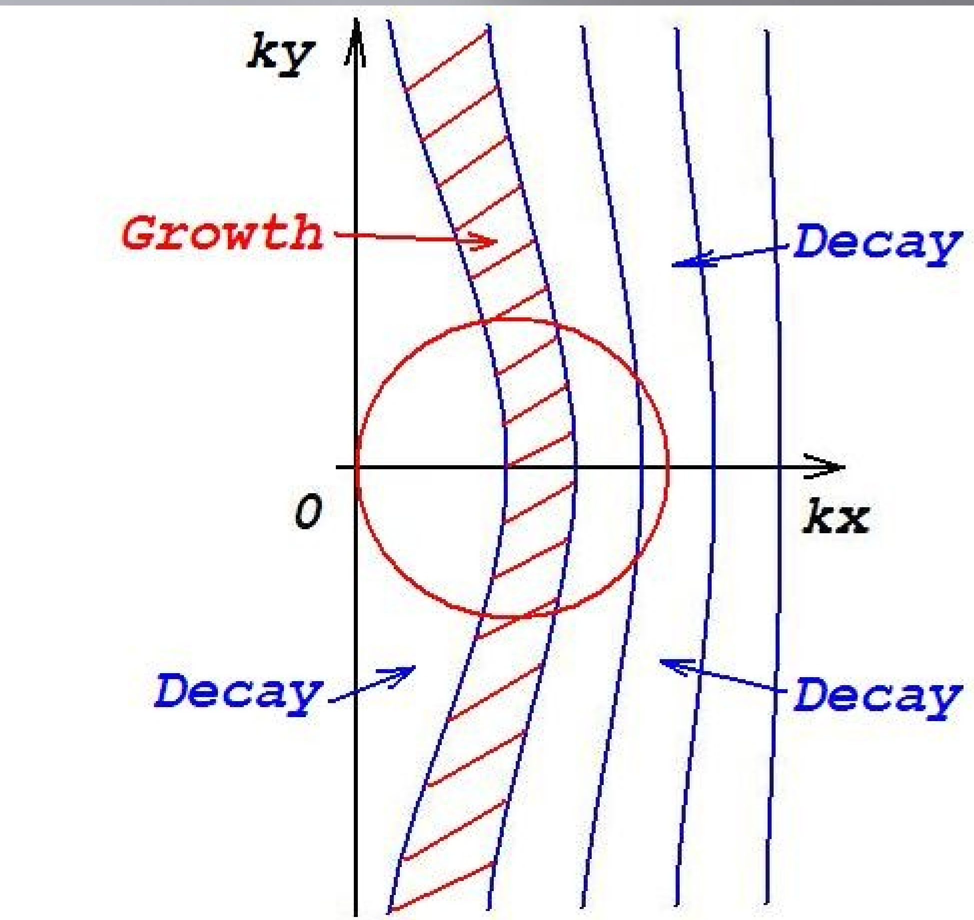
$$\frac{\partial n_k}{\partial t} = \frac{\partial \Omega_k}{\partial k_x} \frac{D}{Dk_y} S \frac{D}{Dk_y} n_k + \gamma_k n_k$$

- Diffusion along curves

$\Omega_k = \omega_k - \beta k_x$ $= conts.$

- S ~ZF intensity

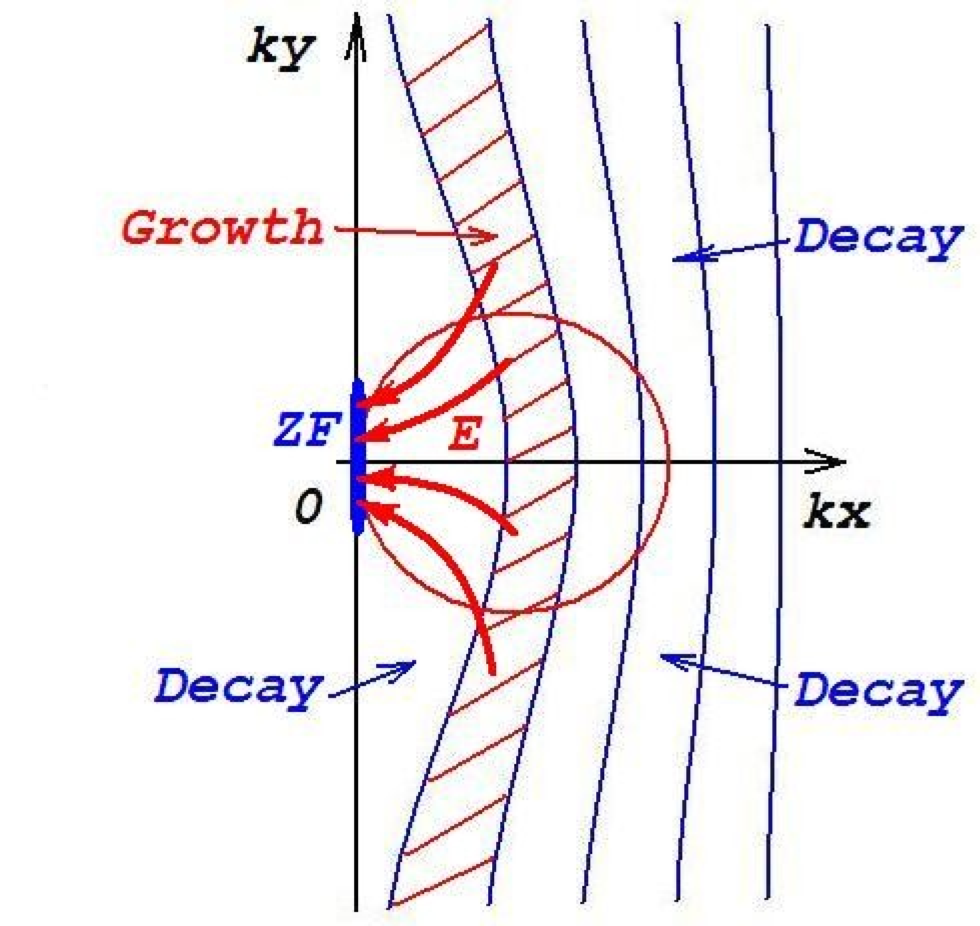
Initial evolution



$$\frac{\partial n_k}{\partial t} = \frac{\partial \Omega_k}{\partial k_x} \frac{D}{D k_y} S \frac{D}{D k_y} n_k + \gamma_k n_k$$

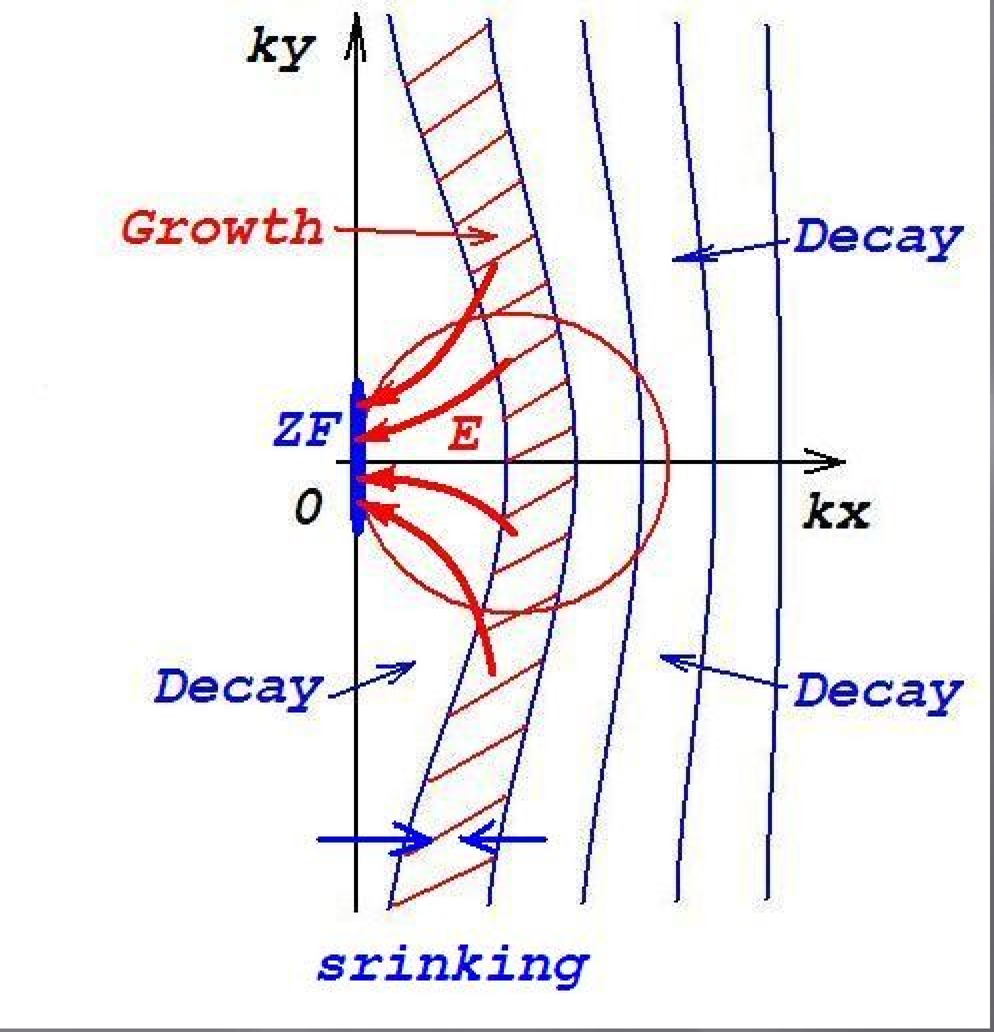
- Solve the eigenvalue problem at each curve.
- Max eigenvalue <0 → spectrum on this curve decay.
- Max eigenvalue >0 → spectrum on this curve grow.
- Growing curves pass through the instability scales

ZF growth



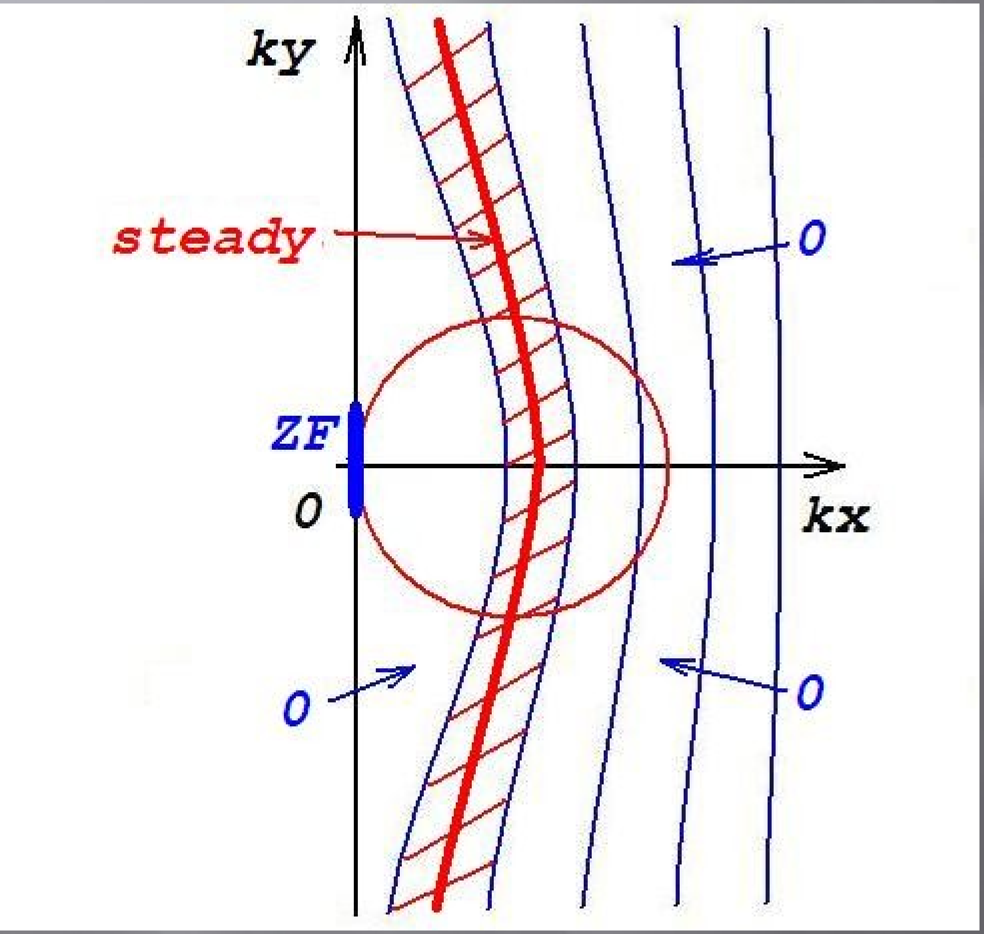
- Waves pass energy from the growing curves to ZF.
- ZF accelerates wave energy transfer to the dissipation scales via the increased diffusion coefficient.

ZF growth



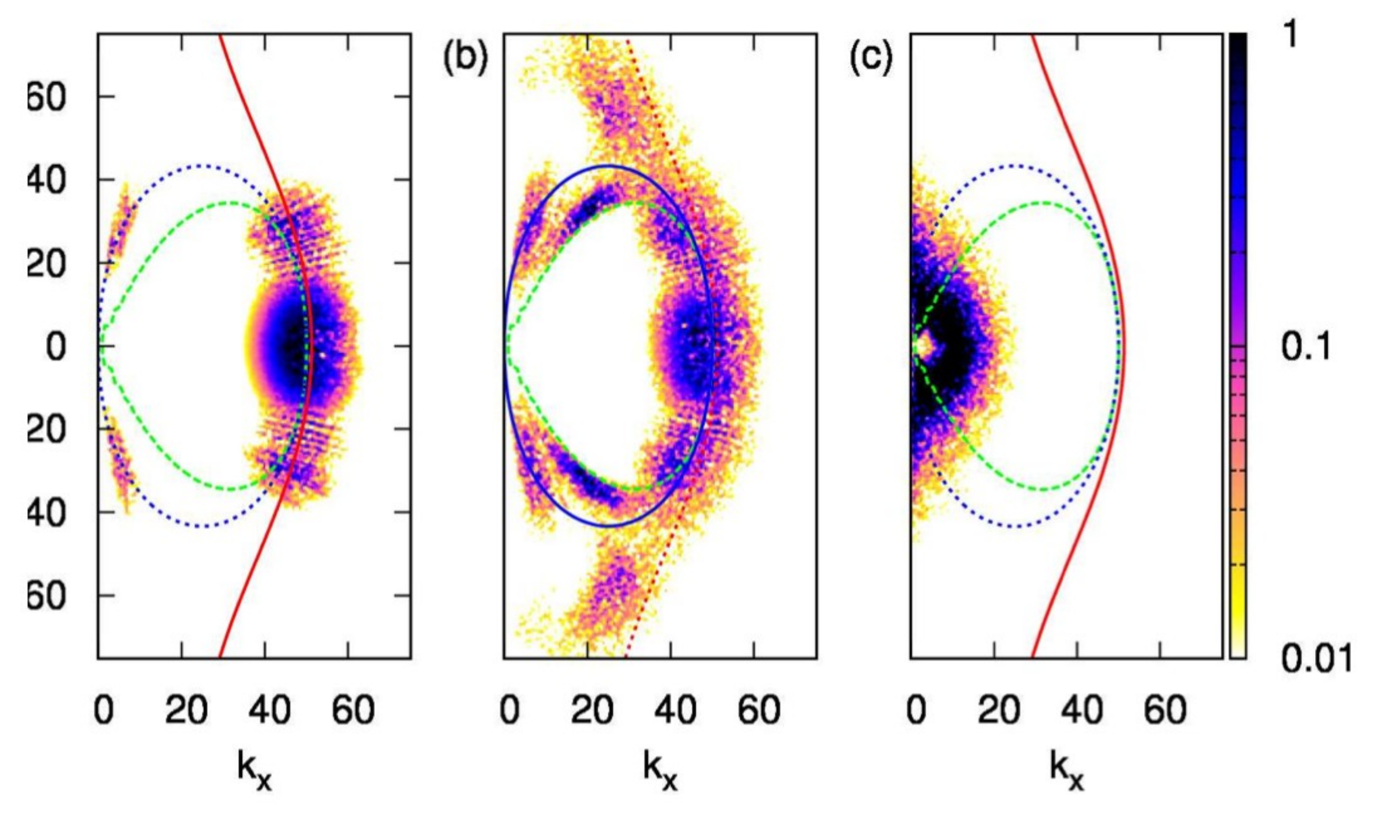
- Hence the growing region shrink.
- Wave Turbulence -ZF loop closed!

Steady state



- Saturated ZF.
- Jet spectrum on a k-curve passing through the maximum of instability.
- Suppressed intermediate scales
- Balanced/correlated turbulence and ZF

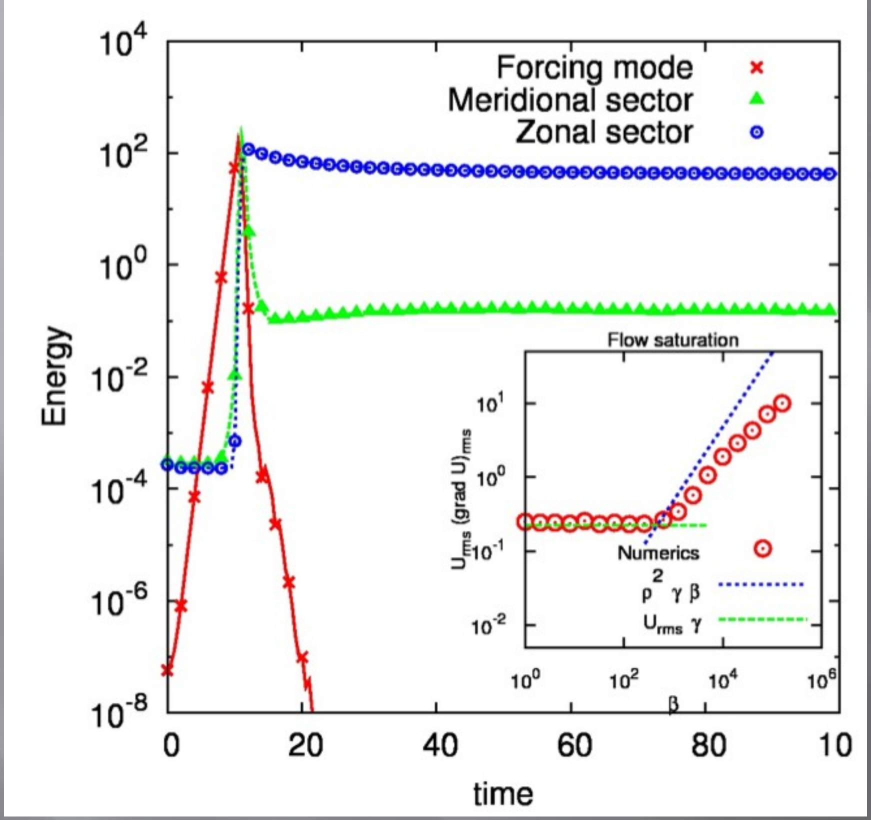
NUMERICS OF INSTABILITY-FORCED CHM



C.Connaughton, SN and B.Quinn, 2010.

- Zonal scales form.
- Small-scale turbulence is suppressed.

NUMERICS OF INSTABILITY-FORCED CHM



C.Connaughton, SN and B.Quinn, 2010.

Evolution in time of energies:
Read – zonal sector,
Green – off-zonal sector;
Blue – instability scales.

- Zonal scales form.
- Small-scale turbulence is suppressed.

Summary

- Importance of resonant wave interactions in QG turbulence
- Generation of zonal jets by local anisotropic cascades and nonlocal mechanisms.
- Quadratic invariants.
- Self-regulating turbulence – zonal jet system
- Continuous spectrum v discrete-wave clusters

Thank you ^^